Comment on ‘Automatic Central Limit Theorems ...’

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1 Better recurrence

Recall that some years ago you were overly generous and credited Nijenhuis and myself with inventing the algorithm for raising a power series to the nth power, that appears in our book *Combinatorial Algorithms*. We pointed out that it was known long before us, and Don Knuth traced it back to the Neanderthals. Anyway that same algorithm gives a somewhat simpler recurrence for your \( f_r(n) \)'s in this paper.

In fact, if

\[
\left( \sum_{r \geq 0} \frac{F_r}{r!} z^r \right)^n = \sum_{r \geq 0} \frac{f_r(n)}{r!} z^r,
\]

then by logarithmic differentiation and matching coefficients one finds that

\[
f_r(n) = \sum_{0 \leq j < r} \binom{n}{r-1} \binom{r-1}{j} F_{j+1} f_{r-1-j}(n),
\]

with \( f_0 = 1, \ f_1 = 0 \), which avoids the extra step of indefinite summation that’s in your present recurrence.