A Maple One-Liner that is a MUCH Better ANSWER than George Andrews' "Explicit" Formula for the Rademacher Coefficients

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Drew Sills and my beloved master, Doron Zeilberger, have just posted (`http://www.math.rutgers.edu/~zeilberg/mamarim/mamarimhtml/hans.html`) a beautiful article refuting ("empirically" but most convincingly) a forty-year-old conjecture of the eminent number theorist Hans Rademacher (that they were hoping that Andrews (Rademacher's academic *Benjamin*) would consider for the PNAS, but he declined, since "empirical" proofs do not count, and possibly also because it didn't mention modular forms or Maass forms). George Andrews himself attempted to take a first step towards proving this conjecture (*Ramanujan J.* **7** (2003) 385-400 ; `http://www.springerlink.com/content/q7g361w42231r247/`), and proved (Theorem 1., p. 388) an *explicit* formula for the so-called $C_{011}(N+1)$. OMG, with "explicit" formulas like this we don't need any implicit formulas. It has $N!$ summands, each of which takes (at least) exponential time to compute!!!

On the other hand, the Maple one-liner code (Maple was readily available in 2003)

```
H:=N -> coeff(taylor(normal(y**N/mul(1 - (1+y)**i, i = 1 ..  N)),y=0,N),y,N-1):
```

enables one to get in 12 seconds the first 70 terms, by typing: `seq(H(i),i=1..70);` . In 51 seconds, one gets that `H(195)` is $77.796670050\ldots$ (a far cry from $-.292927\ldots$, conjectured by Rademacher to be the limit of $H(n)$ as $n \to \infty$).

Using more clever ideas, Sills and Zeilberger get many more terms, and were able to make a *precise* conjecture about the sequence $\{H(n)\}$ (and many related sequences), but the above one-liner would have sufficed to show that the Rademacher conjecture is (almost certainly) false.

I don't know if the above Maple one-liner qualifies as an "explicit formula", but a *formula* is an algorithm, and Andrews' "formula" is a *very lousy* algorithm. See Herb Wilf's classic "*What is an answer*" (`http://www.jstor.org/pss/2321713`) for other examples.

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Exclusively published in: `http://www.math.rutgers.edu/~zeilberg/pj.html` . Oct. 23, 2011.