

Highlights from the First 400,000 Sequences in the OEIS

**Experimental Math Seminar,
Rutgers University, September 10 2026**

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OUTLINE

The OEIS today; nominations open for A400000

Year Extremal with respect to:

- 500 BCE Differences are squares, most wanted proof
- 300 BCE Most helpful AI contribution
- 1670 Gregory series; best postage stamps
- 1859 Most potentially rewarding
- 1894 Funniest (i)
- 1902 Most spectacular geometry
- 1944 Longest finite sequences
- 1956 Shortest sequence
- 1973 Most appropriate for aliens
- 1987 Most remarkable growth
- 1994 Most wanted primes
- 2004 Best unsolved conjecture
- 2013 Funniest (ii)
- 2022 Best carpet design
- 2023 Scariest
- 2026 Most amazing: Red & Black Knights

The OEIS today

is in very good hands! Russ Cox is President, Sean Irvine is Managing Editor, and Robert Roy Price is Treasurer.

There are many active editors, including: Joerg Arndt, Jon Awbrey, Michael Branicky, David Corneth, Susanna Cuyler, Harvey Dale, Michael De Vlieger, Amiram Eldar, Felix Fröhlich, Charles Greathouse, Maximilian Hasler, Alois Heinz, Andrew Howroyd, Robert Israel, Vaclav Kotesovec, Peter Luschny, Bob Lyons, Seiichi Manyama, Michel Marcus, Hugo Pfoertner, Jianing Song, Stefano Spezia, Hugo van der Sanden, Pontus von Brömssen, Jinyuan Wang, Paolo Xausa, Jason Yuen, Andrei Zabolotskii, and dozens more.
(This list includes several Trustees.)

The new Sequence Fans Mailing List (<https://groups.google.com/g/seqfan>) is flourishing, with 435 members.

My role as Chairman leaves me free now to help out with editing, study individual sequences, collaborate with friends, write articles, and make videos!

Statistics as of September 7, 2026, from Sean Irvine:
Total entries: 399054 (+901 in past month)
Registered users: 16305 (+360 in past month)
Mean submissions stack size: 301
Citations of OEIS: 13000

Nominations open for A400000

[Written Sept. 7 2026]

Nominate a sequence (it could be your own, or someone else's) to get the A-number A400000.

**Sean Irvine is about to post the rules on the SeqFan List and on the main OEIS Wiki page:
https://oeis.org/wiki/Main_Page**

**The deadline will probably be October 16 2026
[The email address for sending nominations is
400k@oeis.org. But please read the rules first!]**

500 BCE

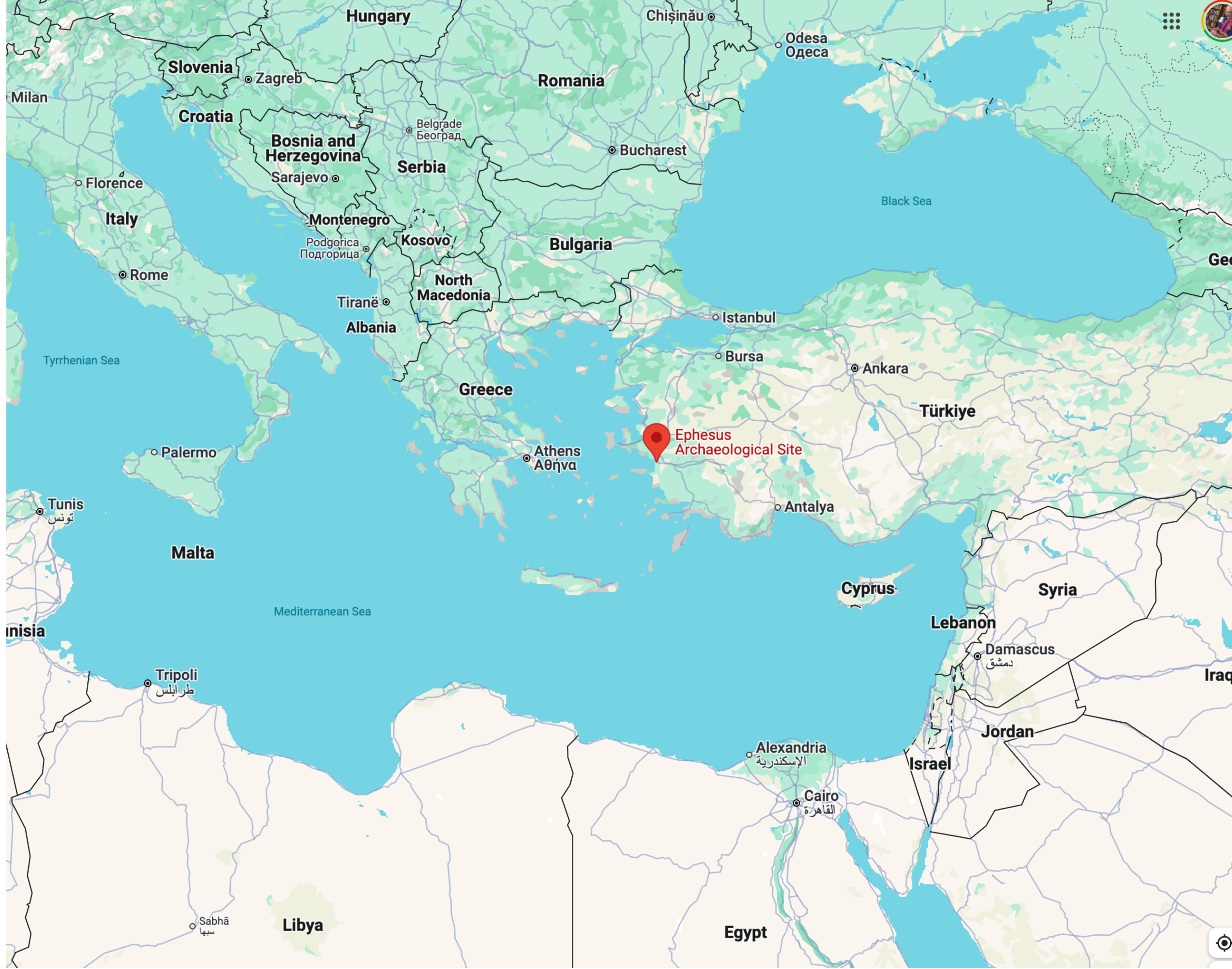
The Heraclitus Transform

**Around 500 BCE, Heraclitus of Ephesus said
“No man can step in the same river twice”.**

**The definitions of many OEIS sequences include the requirement that
“The sequence may not include the same number twice”
which we will call the **Heraclitus Rule!****

**A fine example (which is unsolved) is the Christmas Tree sequence [A377091](#) of Rémy Sigrist
(see following slides).**

The Heraclitus Transform

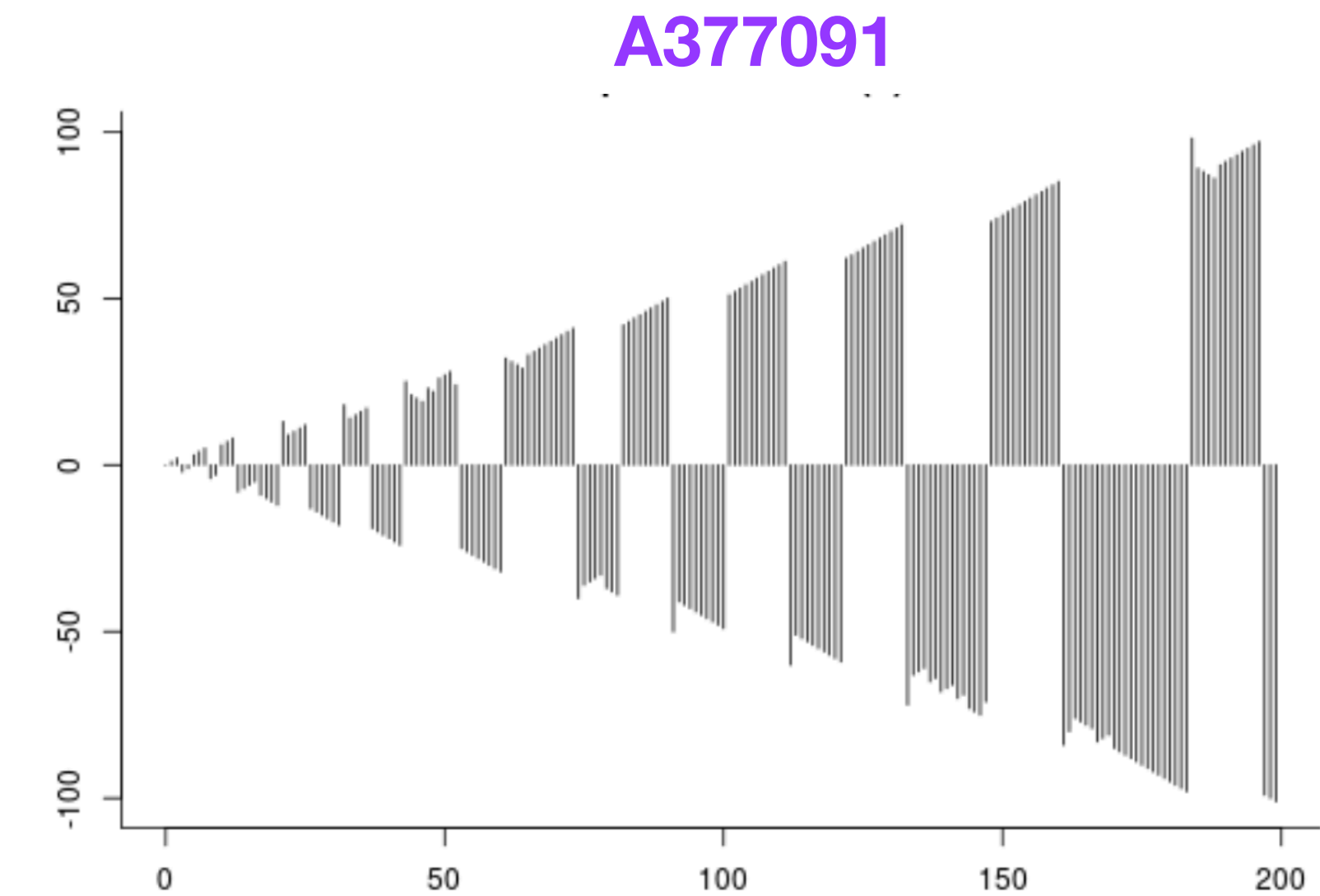
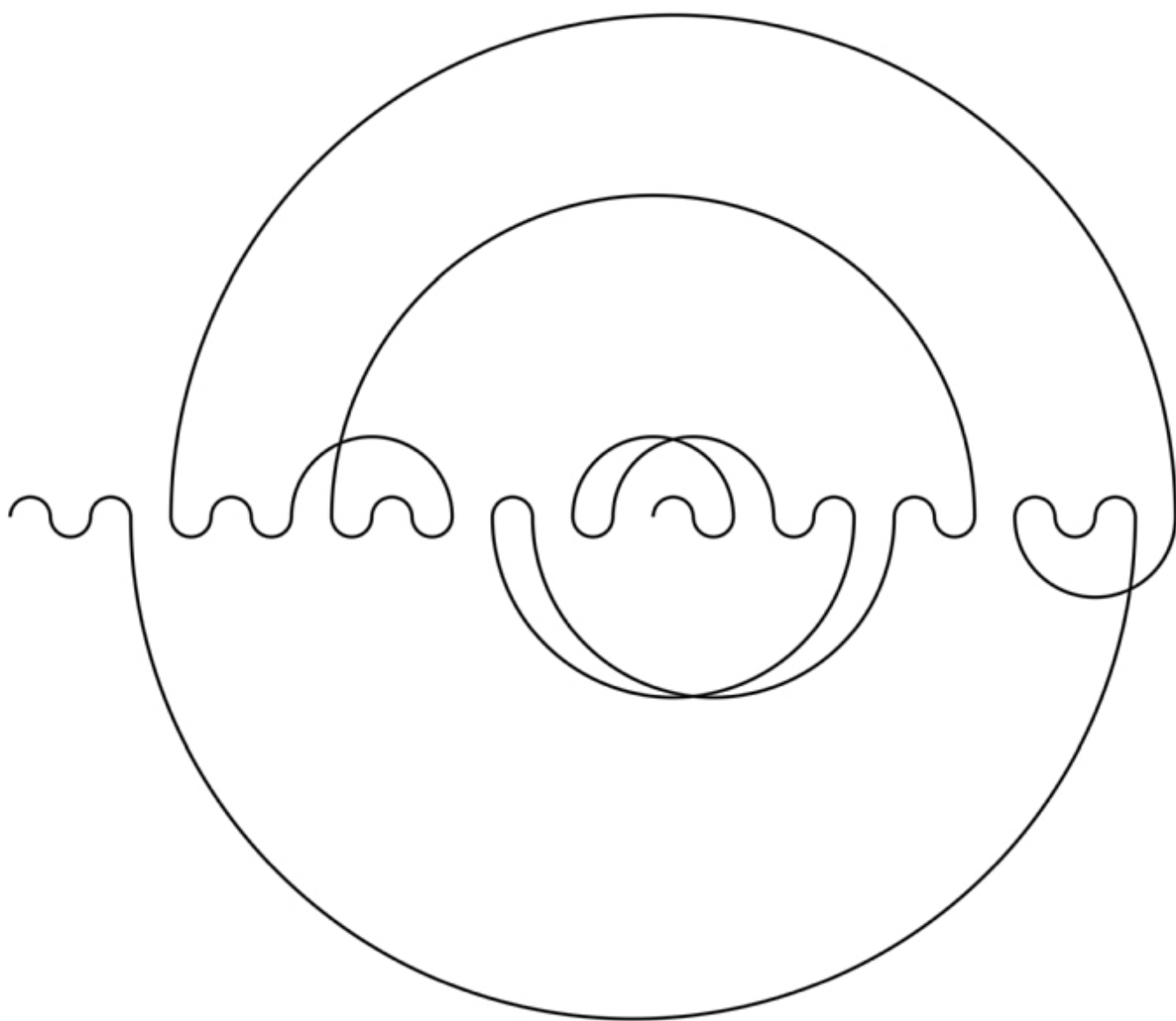


The Heraclitus Transform (cont.)

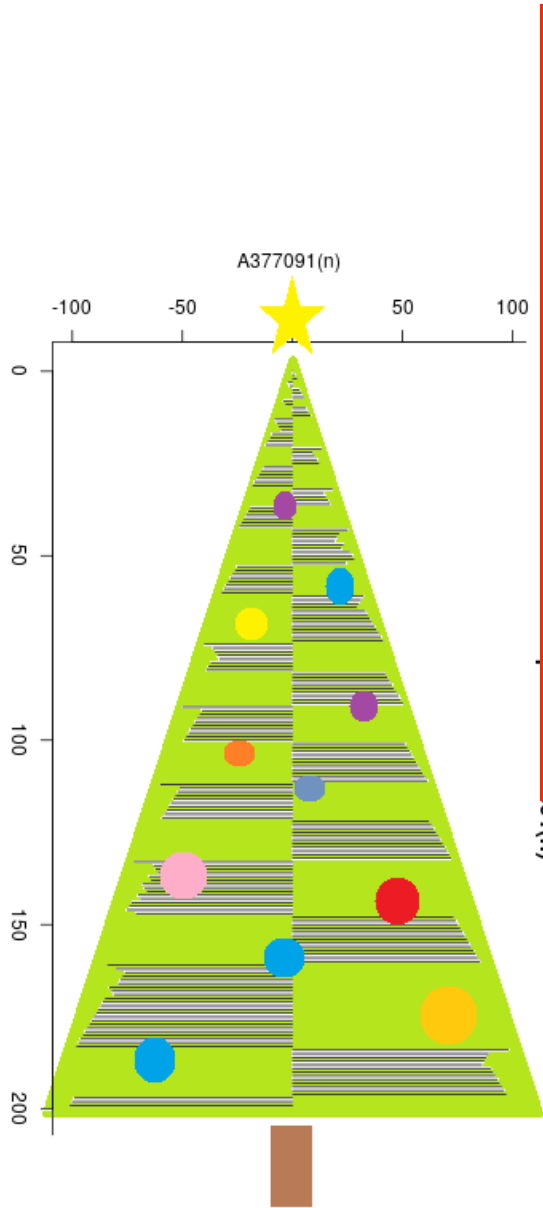
The Christmas Tree sequence of Rémy Sigrist, [A377091](#):

Start at 0 and walk along the number line.
Make steps of sizes 1, 4, 9, 16, etc (the squares) left or right.
You must move as close to 0 as possible
(if there is a choice, pick the positive number)
except that Heraclitus Rules: no number may be repeated

0, 1, 2, -2, -1, 3, 4, 5, -4, -3, 6, 7, 8, -8, -7, -6, -5, -9, -10, -11, -12, 13, 9, 10, 11, ...



This is the Heraclitus Transform HT(Squares)



OPEN PROBLEM:
Show that every number,
positive or negative,
will eventually appear!
Show there are
no missing numbers.
Most wanted proof!

30+ derived seqs., many conjectured
properties, many contributors,
Especially. Paolo Xausa

July 2026: Undergrad sent
AI "proof", which was garbage.

The Heraclitus Transform (cont.)

Alternative version of previous slide for Al-Khwarizmi Seminar,

Tunis, Tunisia, May 2025 (not given).

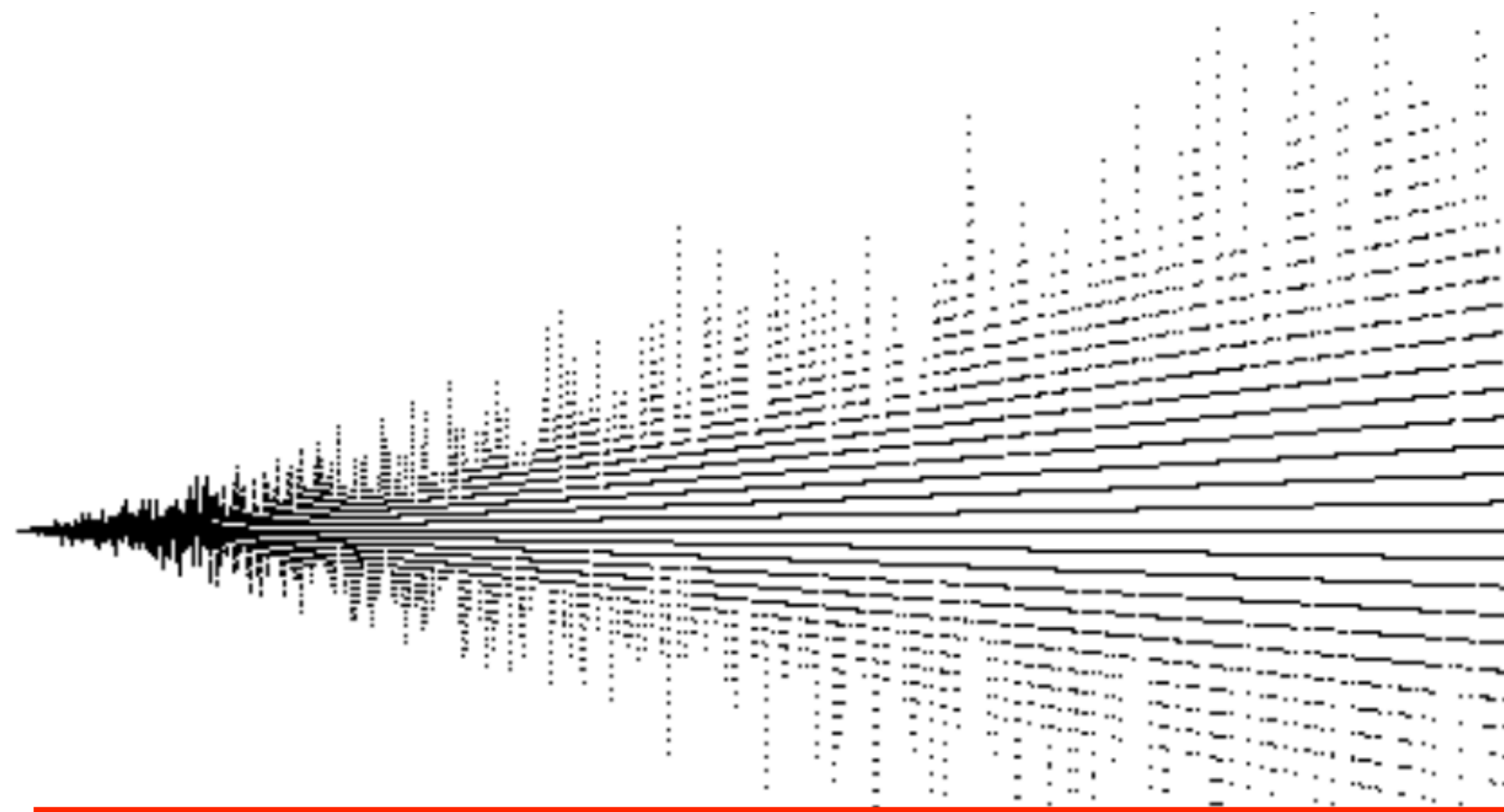
Replace Christmas Tree with Cedar of Lebanon:



The Heraclitus Transform (cont.)

Another example: the Heraclitus Transform of the primes, [A377090](#)

0, 2, -1, 1, -2, 3, -4, -6, -3, 4, 6, -5, -7, -9, 8, 5, 7, 9, -8, -10, -12, 11, 13, 10, 12, ...



SAME (OPEN) PROBLEM:
Show that every number,
positive or negative,
will eventually appear!

The Heraclitus Conjecture. Let S = set of allowed step sizes. Can assume steps are positive, distinct, and arranged in increasing order.

Conjecture: For such an S , $HT(S)$ contains every integer iff S is infinite and $\gcd(S) = 1$.

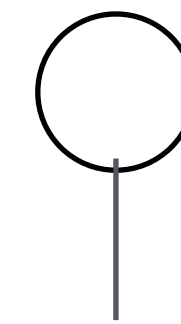
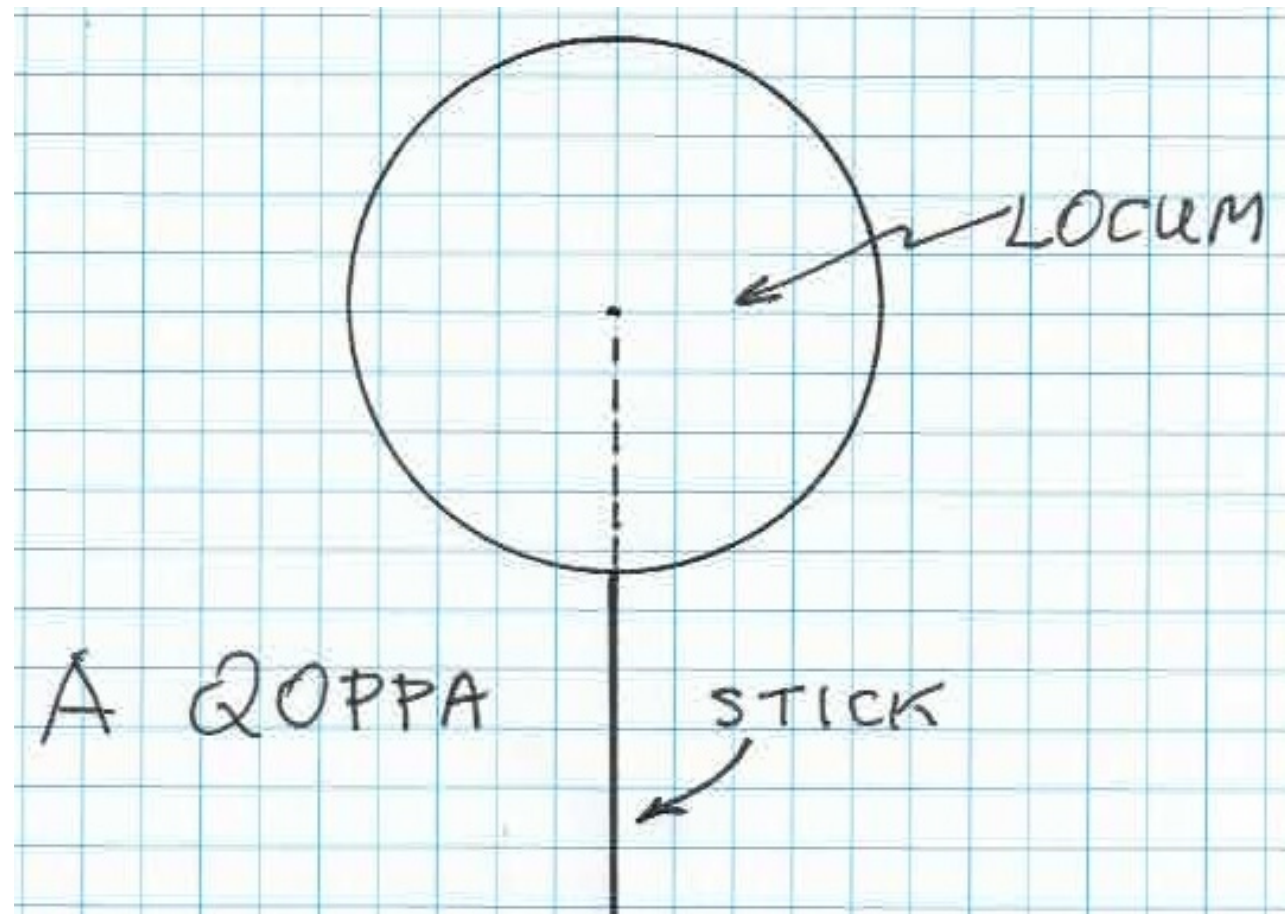
S must be infinite, or you get stuck on either the positive or negative side. Obviously $\gcd$ must be 1.
But is this enough to show that you hit every number? My current guess is Not!

It would be nice to have a counter-example!

300 BCE

Euclid on the Beach

To keep the children quiet, Mrs Euclid would make a sweet out of lokum:

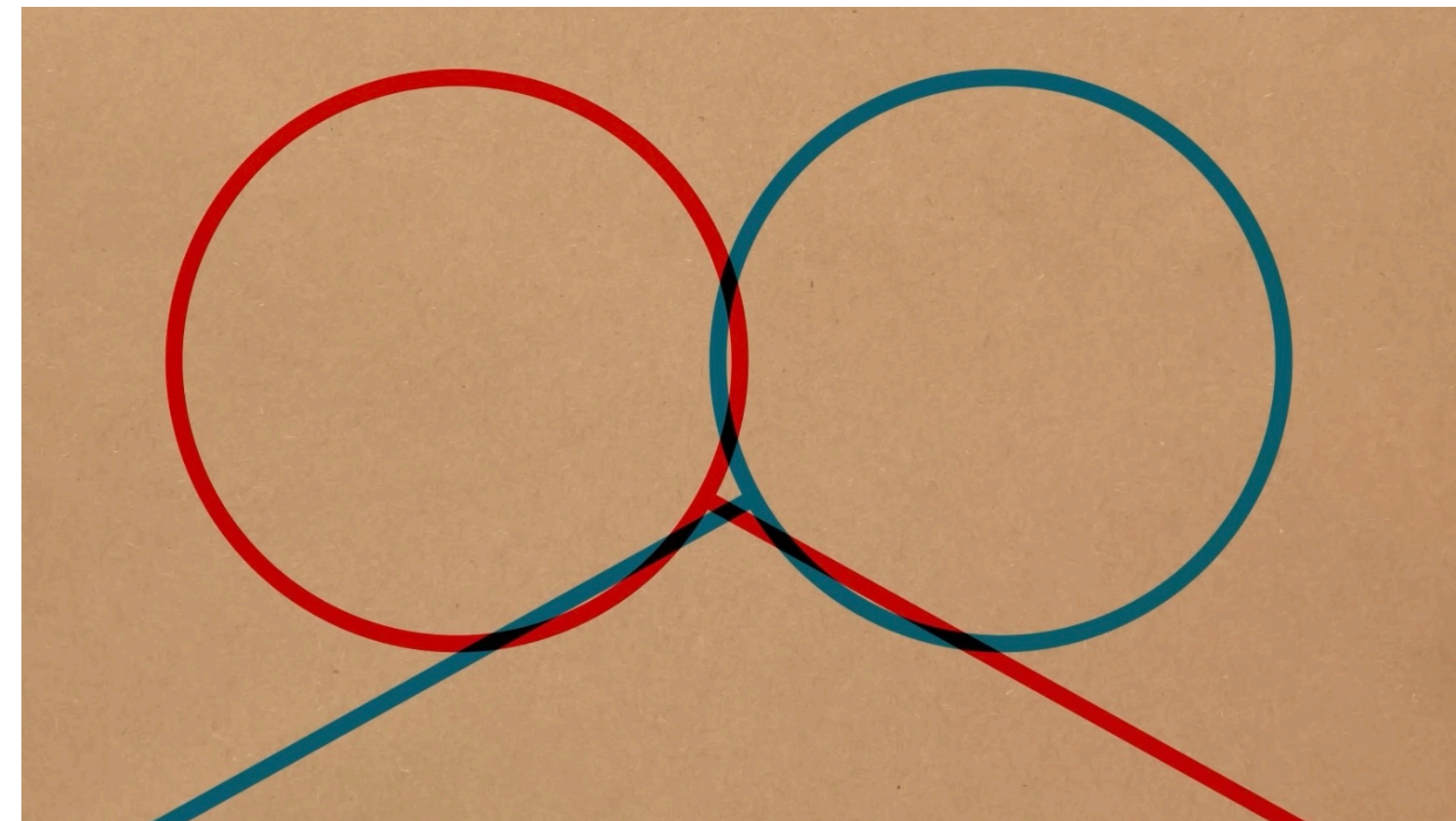


The archaic Greek letter Qoppa.

Over the centuries, because of the shift in consonants,
the sweet changed from a qoppa to a poppa,
and then to a lollipoppa,
and today it is called a lollipop.

Euclid: A qoppa divides the beach into two regions
How many regions can you get with two qoppa?

**Answer: 10 regions, by making the
two qoppa intersect in 7 points.**



Euclid on the Beach (cont.)

What about 3, 4, 5, ... qoppa?
Once you have done 2, 3 qoppa is easy:

[We now skip ahead to 2025]
Euler's formula says that maximizing the number (R) of regions
is the same as maximizing the number (V) of intersections:

Proof: Assume no triple intersections, etc. Let R = regions, V = vertices where distinct qoppa cross. With n qoppa Euler gives $R = V + n + 1$. Two qoppa can intersect in at most 7 vertices. So $R \leq 7n(n-1)/2 + n + 1$ Equality holds iff $n \leq 3$.

(See "Cutting a pancake with an exotic knife",
Cutler-Karlsson-NJAS, for details of proof.)

**n = 3: V = 21 intersections,
R = 21 + 3 + 1 = 25 regions**

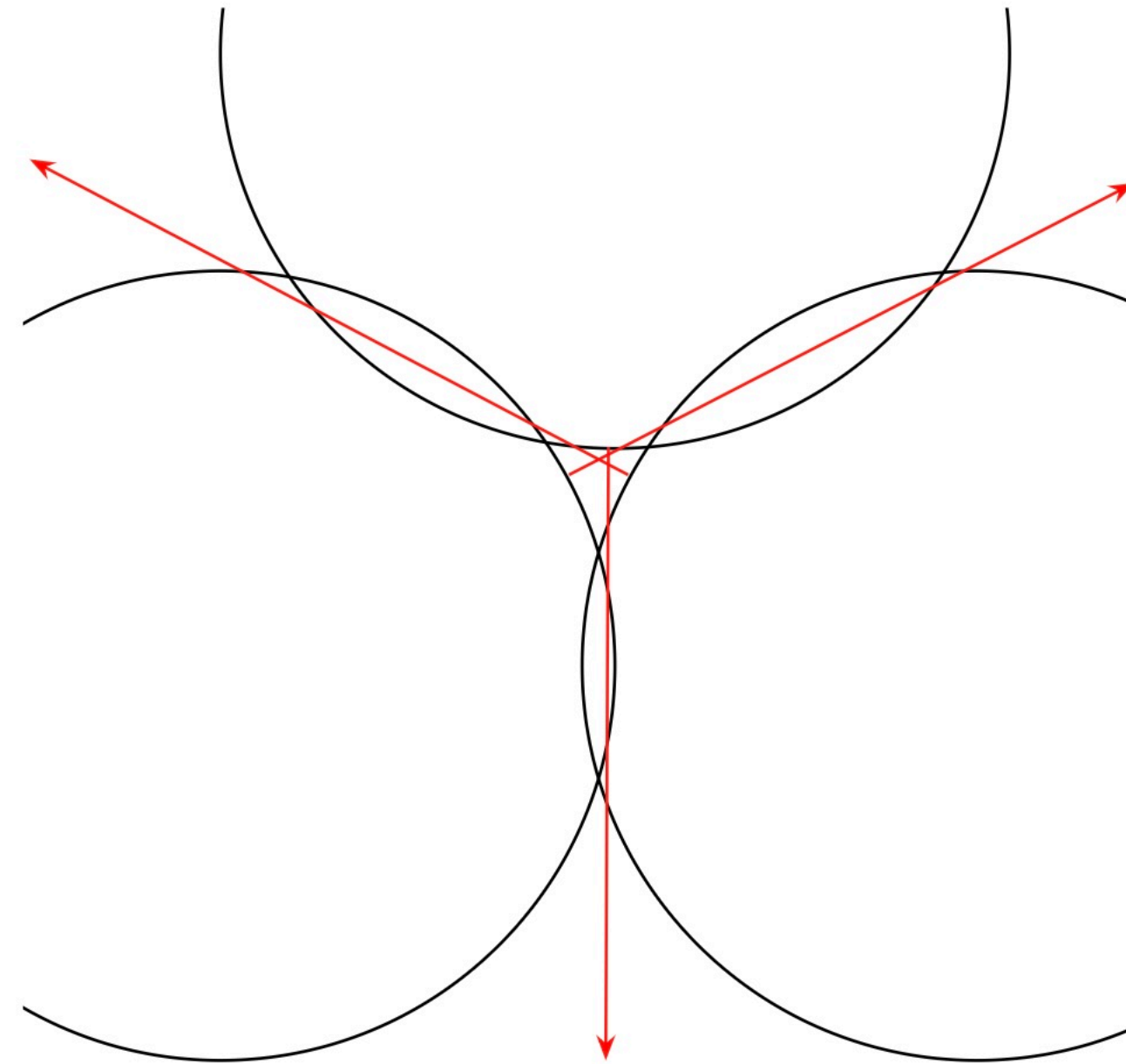
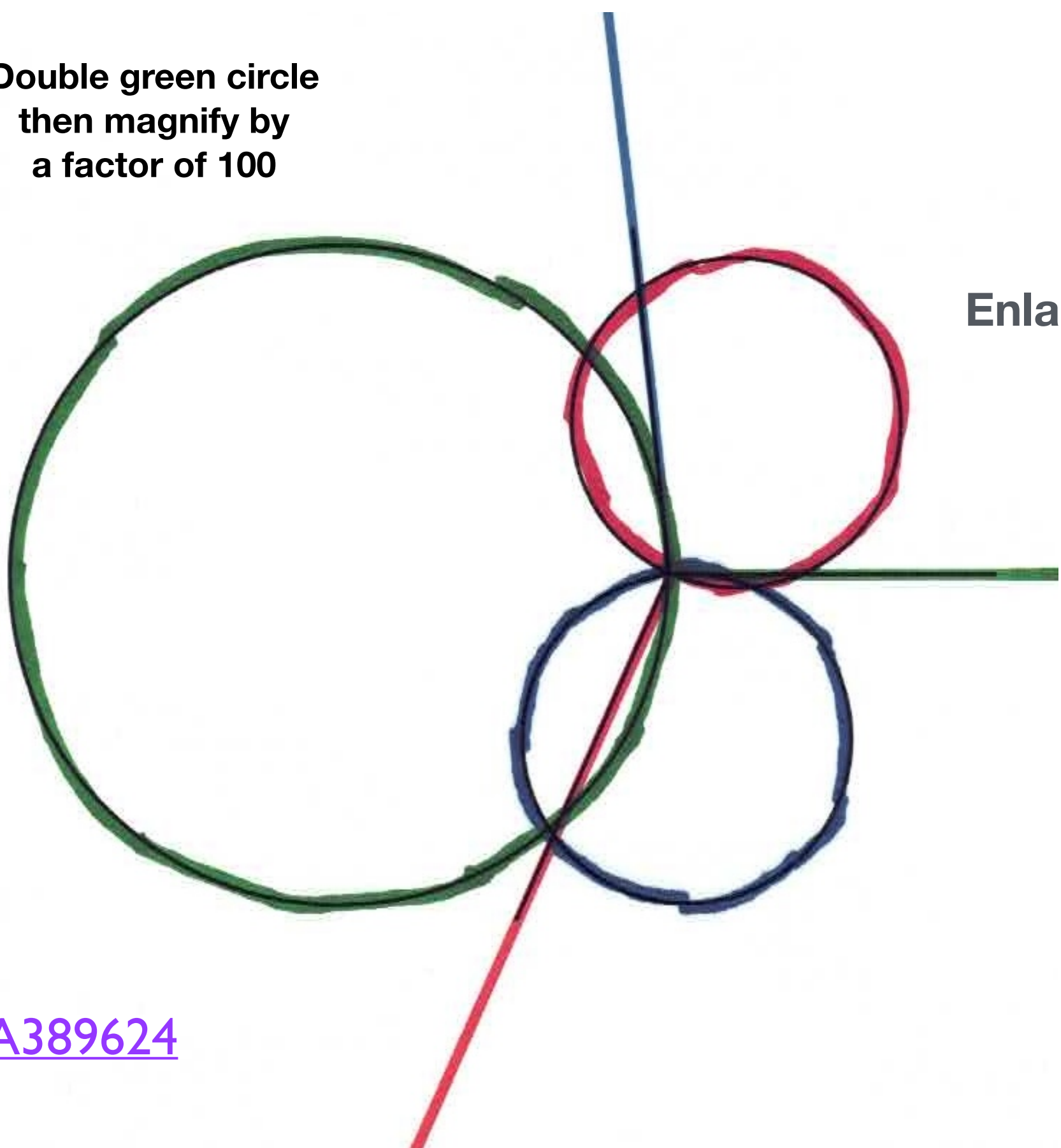


Figure 40: With each pair intersecting in 7 points, three lollipops can divide the plane into 25 regions.

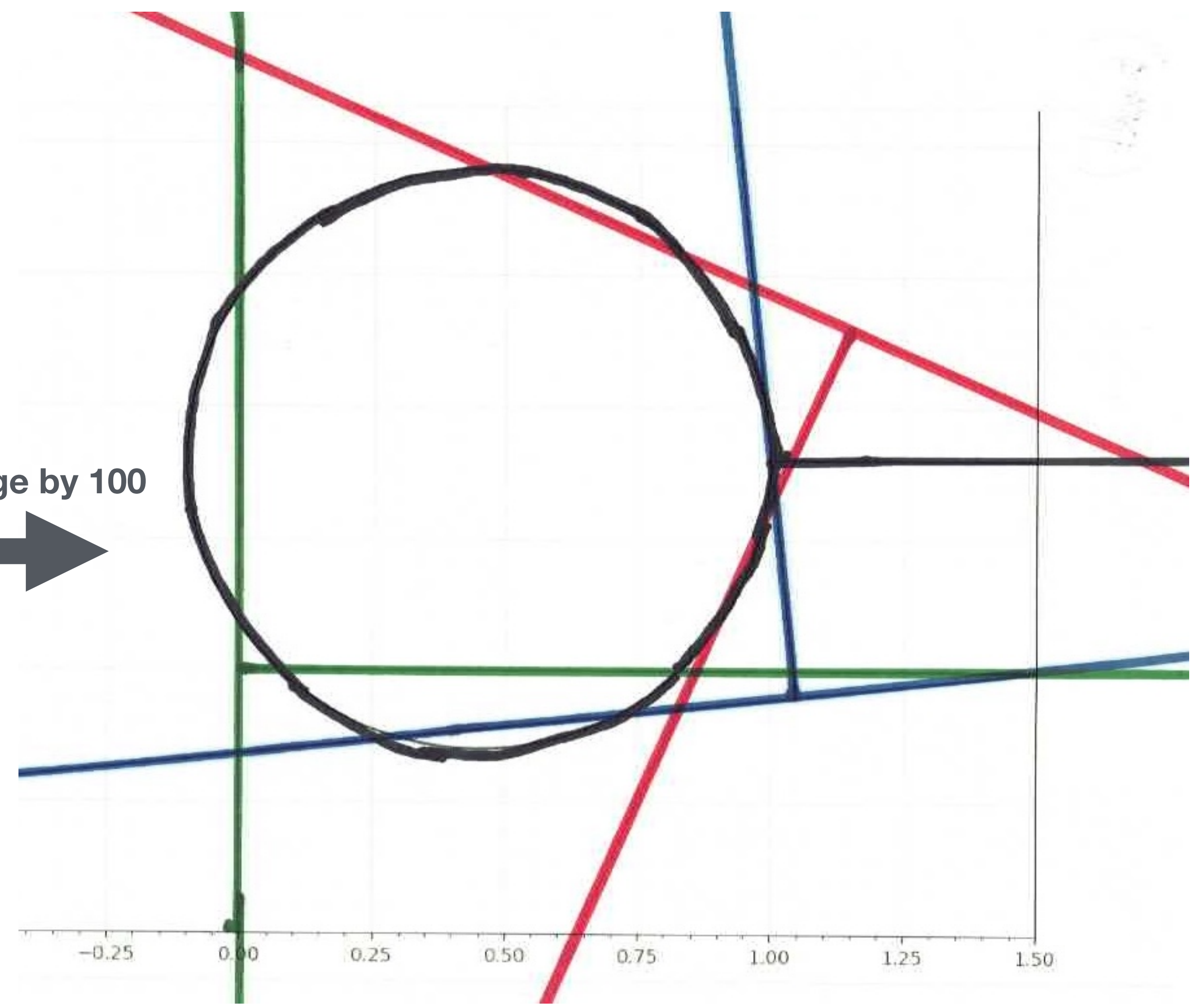
Euclid on the Beach (cont.)

**n = 4: V = 40 intersections,
R = 40 + 4 + 1 = 45 regions.
Discovered and proved optimal
by Jonas Karlsson on
December 25-26 2025.**

Double green circle
then magnify by
a factor of 100



Enlarge by 100
➔



Closeup: green, red, blue circles now look like straight lines.
Place black circle as shown.
Black circle also present in left picture, but is too tiny to be seen.

Euclid on the Beach (cont.)

As of March 2026

[A389624](#)

n	0	1	2	3	4	5	6	7	8
R	1	2	10	25	45	71-72	104-106	142-146	186-193
(Cutler-Karlsson-Sloane, March 2026 (J. I. S., 2026))									

R	1	2	10	25	45	71	104	142	186
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Tavor Gochman,
Siddhartha Mahajan
& Paras Chopra,
Matthais Paulsen

(AI from Tavor Gochman, Siddhartha Mahajan &
Paras Chopra, analysis from Matthais Paulsen)
and (unconfirmed AI) exact formula

$$R = F(n) + 2n^2 - n + 1$$

where

$$F(n) = \max_{\substack{m_1 + m_2 + m_3 + m_4 = n \\ m_i > 0}} \left[3 \sum_{i < j} m_i m_j - 2 \min_{i < j} m_i m_j \right]$$

1670

Mayotte (Fr): Philippe Deléham, 1997

Gregory coefficients^(*) (1670)

$$1 + \sum_{n=1}^{\infty} G_n x^n = \frac{x}{\log(1+x)}$$

$$G_n = \frac{1}{2}, -\frac{1}{12}, \frac{1}{24}, -\frac{19}{720}, \dots$$

$$\text{Deléham: } a_n = n!(n+1)! |G_n|$$
$$= 1, 1, 6, 76, 1620, 51780, \dots$$

$$\text{Properties: } \sum |G_n| = 1, \quad \sum \frac{|G_n|}{n} = \text{Euler } \gamma$$

Philippe Deléham : over 5000 mentions in OEIS (1997-2024)

[A002206/A002207 \(G_n\)](#), [A009763 \(a_n\)](#)

(*) Used in Gregory's formula, earliest formula for numerical integration. (Not to be confused with Gregory's series.)

Mayotte (cont.)

Philippe Deléham's first letter,
from Mayotte, later messages
from Lifou, New Caledonia.
Author of 1100 sequences,
thousands of comments

Monsieur DELEHAM Philippe

BP 29 Coconi

97670 Ouangani

MAYOTTE

(FRANCE)

Coconi le 14/04/97

A9763

à Monsieur N.J.A. SLOANE

Monsieur

Je me suis procuré votre livre "The encyclopedia of integer sequences".

Je suis assez étonné de ne pas y avoir trouvé la suite 1, 1, 6, 76, 1620, ... liée à la constante d'Euler $\gamma = 0,577215664 \dots$ (M 3755)

en effet
$$\gamma = \frac{1}{1 \cdot 1! \cdot 2!} + \frac{1}{2 \cdot 2! \cdot 3!} + \frac{6}{3 \cdot 3! \cdot 4!} + \frac{76}{4 \cdot 4! \cdot 5!} + \frac{1620}{5 \cdot 5! \cdot 6!} + \dots$$

En utilisant le théorème de Raabe

$$\int_x^{x+1} \log \Gamma t \, dt = x (\log x - 1) + \frac{1}{2} \log(2\pi) \quad \text{il vient :}$$

Whittaker and Robinson,
The Calculus of Observations,
4th ed., 1943:

James Gregory's Formula for
Numerical Integration

This is the Euler-Maclaurin formula with a remainder-term expressing the error committed when the series is truncated at the terms involving the third derivatives. By proceeding in the same way we can find the error committed in truncating the Euler-Maclaurin expansion at any assigned term.

If $f(x)$ is a polynomial, the Euler-Maclaurin series terminates and is exact.

72. Gregory's Formula of Numerical Integration.—The earliest formula of numerical integration to be discovered was one in which the correction terms were expressed in terms of differences instead of derivatives. This formula, which is of great use in practice, may be derived from the Euler-Maclaurin formula in the following way:

In the Euler-Maclaurin formula (see A060055)

$$\frac{1}{w} \int_a^{a+w} f(x) dx = \left(\frac{1}{2} f_0 + f_1 + f_2 + \dots + f_{r-1} + \frac{1}{2} f_r \right) - \frac{w}{12} (f'_r - f'_0) + \frac{w^3}{720} (f'''_r - f'''_0) - \frac{w^5}{30240} (f^{(5)}_r - f^{(5)}_0) + \dots$$

let us substitute for the derivatives from the equations

$$\begin{aligned} wf'_0 &= \Delta f_0 - \frac{1}{2} \Delta^2 f_0 + \frac{1}{6} \Delta^3 f_0 - \frac{1}{24} \Delta^4 f_0 + \frac{1}{120} \Delta^5 f_0 - \dots \\ wf'_r &= \Delta f_{r-1} + \frac{1}{2} \Delta^2 f_{r-2} + \frac{1}{6} \Delta^3 f_{r-3} + \frac{1}{24} \Delta^4 f_{r-4} + \frac{1}{120} \Delta^5 f_{r-5} + \dots \\ w^2 f''_0 &= \Delta^2 f_0 - \frac{1}{2} \Delta^3 f_0 + \frac{1}{4} \Delta^4 f_0 - \dots \\ w^2 f''_r &= \Delta^2 f_{r-2} + \frac{1}{2} \Delta^3 f_{r-3} + \frac{1}{4} \Delta^4 f_{r-4} + \dots \\ w^3 f'''_0 &= \Delta^3 f_0 - \dots \\ w^3 f'''_r &= \Delta^3 f_{r-3} + \dots \\ w^4 f^{(4)}_0 &= \Delta^4 f_0 - \dots \\ w^4 f^{(4)}_r &= \Delta^4 f_{r-4} + \dots \end{aligned}$$

etc.

It may be remarked that the formulae for the derivatives of f_0 are given in § 35, and that the formulae for the derivatives of f_r may be obtained by forming a difference-table in which the entries are reversed in order, i.e. $f_r, f_{r-1}, f_{r-2}, f_{r-3}, \dots, f_1, f_0$, and applying the equations of § 35.

Thus we obtain

$$\begin{aligned} \frac{1}{w} \int_a^{a+w} f(x) dx &= \left(\frac{1}{2} f_0 + f_1 + f_2 + \dots + f_{r-1} + \frac{1}{2} f_r \right) - \frac{1}{12} (\Delta f_{r-1} - \Delta f_0) \\ &\quad - \frac{1}{24} (\Delta^2 f_{r-2} + \Delta^2 f_0) - \frac{19}{720} (\Delta^3 f_{r-3} - \Delta^3 f_0) - \frac{3}{160} (\Delta^4 f_{r-4} + \Delta^4 f_0) \\ &\quad - \frac{863}{60480} (\Delta^5 f_{r-5} - \Delta^5 f_0) - \frac{275}{24192} (\Delta^6 f_{r-6} + \Delta^6 f_0) - \dots \end{aligned}$$

The coefficients are $\frac{A002206}{A002207}$

The coefficients are G_n
= A002206/A002207,
which are essentially the
 a_n of Deléham's letter

PAR AVION



Monsieur N. J. A. SLOANE

Room 2C-376

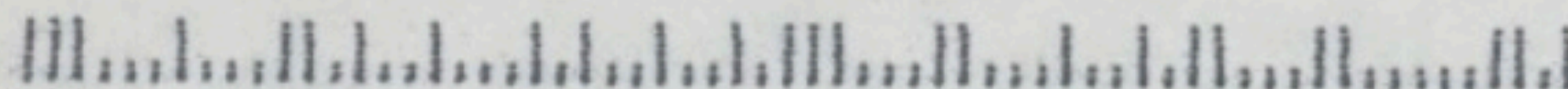
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Center

AT & T Bell Labs-

600 Mountain Avenue

Murray Hill

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MAYOTTE (FRANCE) -



Noor Akhoun

Copyright Noor Akhoun



Most Potentially Rewarding Sequence

A057641 $a(n) = \left[H(n) + e^{H(n)} \log H(n) \right] - \sigma(n)$
 where $H(n) = \sum_{k=1}^n \frac{1}{k}$, $\sigma(n) = \sum_{d|n} d$; $n \geq 1$.

0, 0, 1, 0, 4, 0, 7, 2, 7, 5, 13, 0, 17, 9, 12, 8, 23, ...

Th. J. C. Lagarias, Am. Math. Monthly, 109 (2002), 534-543:
 $a(n) \geq 0$ for all n iff Riemann Hypothesis is true.

Extending work of

G. Robin, J. Math. Pures Appl., 63 (1984), 187-213.

See also A079256, A259632.

The Kubelsky Sequence

Jack Benny's reported age, sampled annually A056064)

1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27,
28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 39, 39, 39, 39, 39, 39, 39, 39, 39, 39, 39,
39, ...

(Benjamin Kubelsky, Feb 14 1894 - Dec 26 1974)

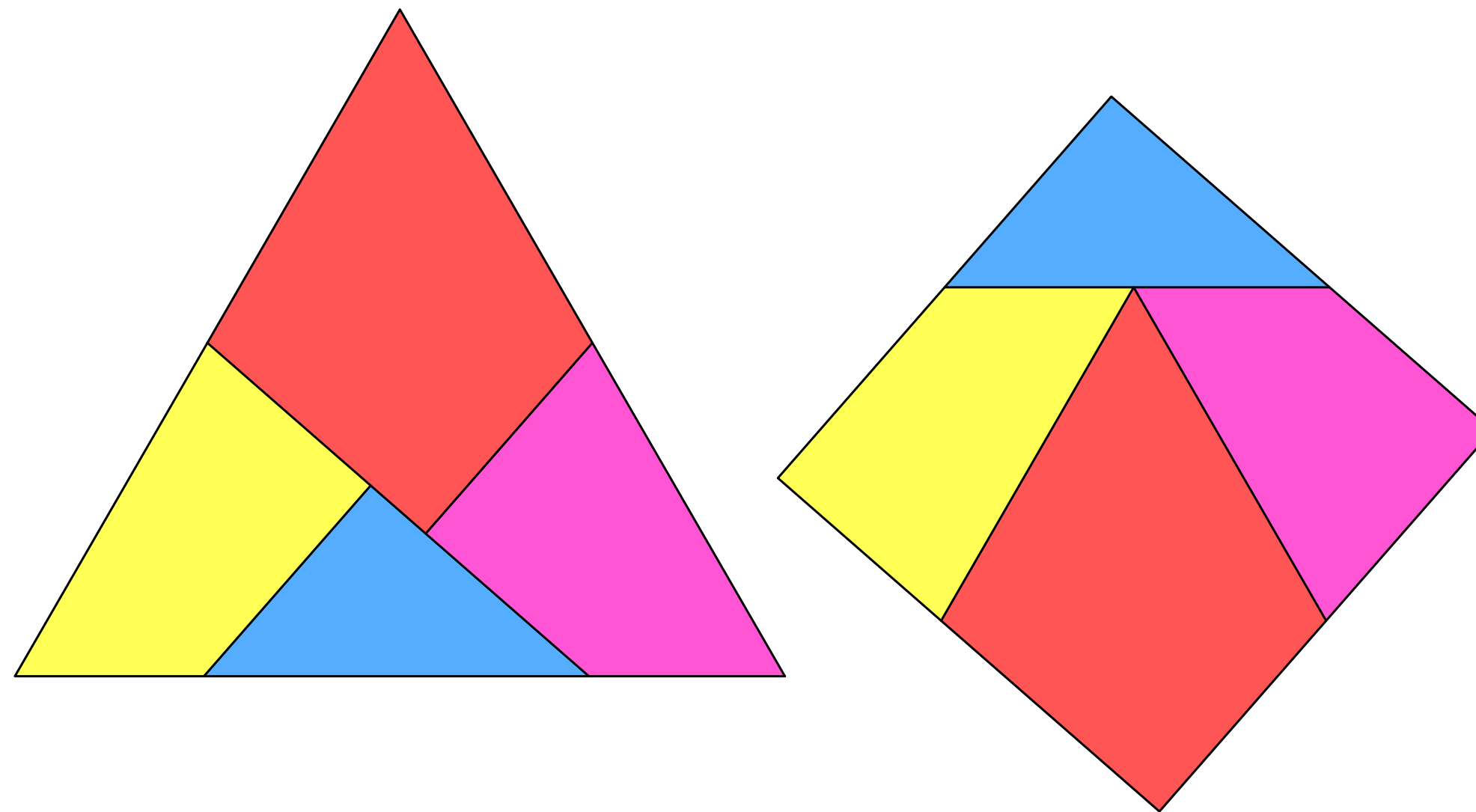
1902

Dissecting Polygons

Henry Dudeney (1902): What is min number of pieces needed to dissect an equilateral triangle to a square?

Cuts must be simple curves, turning over is allowed.

Open for 120 years: show answer is $s(3) = 4$:



Erik D. Demaine, Tonan Kamata, and Ryuhei Uehara, [Dudenay's Dissection is Optimal](#), arXiv, 2025.
(Shows 4 pieces optimal if only allow straight cuts)

Dissecting polygons (cont.)

Let $s(n)$, $r(n)$, $q(n)$ = min number of pieces needed to dissect regular n -gon to a square, rectangle, or monotile

n -gon to square

n -gon to rectangle

n -gon to monotile

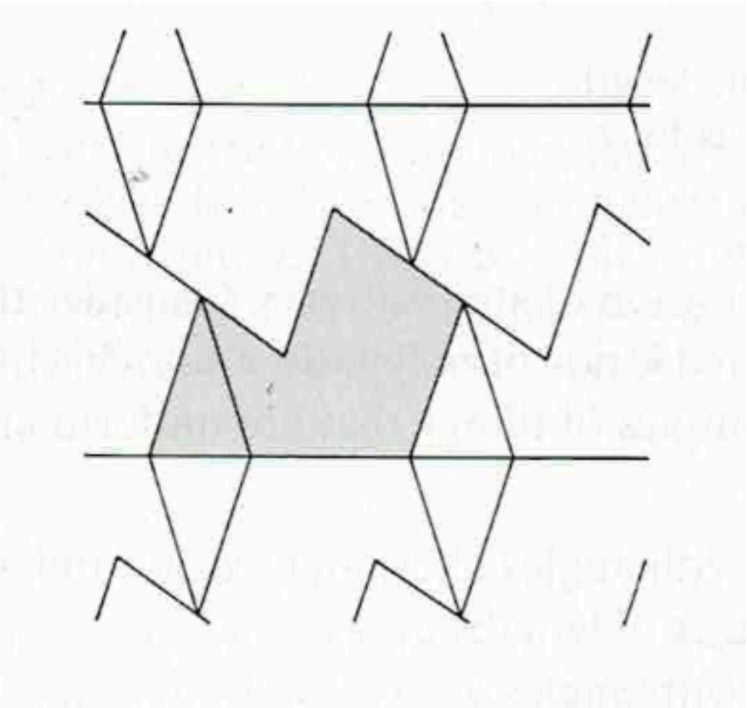
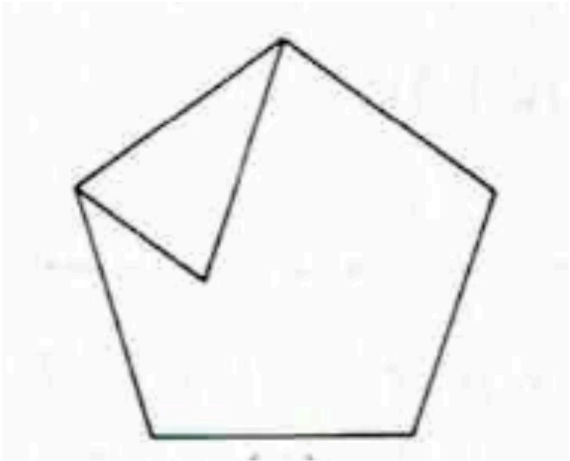
$n =$	3	4	5	6	7	8	9	10	11	12
$s(n)$ $\leq$	4	1	6	5	7	5	9	7	10	6
$r(n)$ $\leq$	2	1	4	3	5	4	7	4	9	5
$q(n)$ $\leq$	1	1	2	1	3	2	3	2	4	3

OEIS

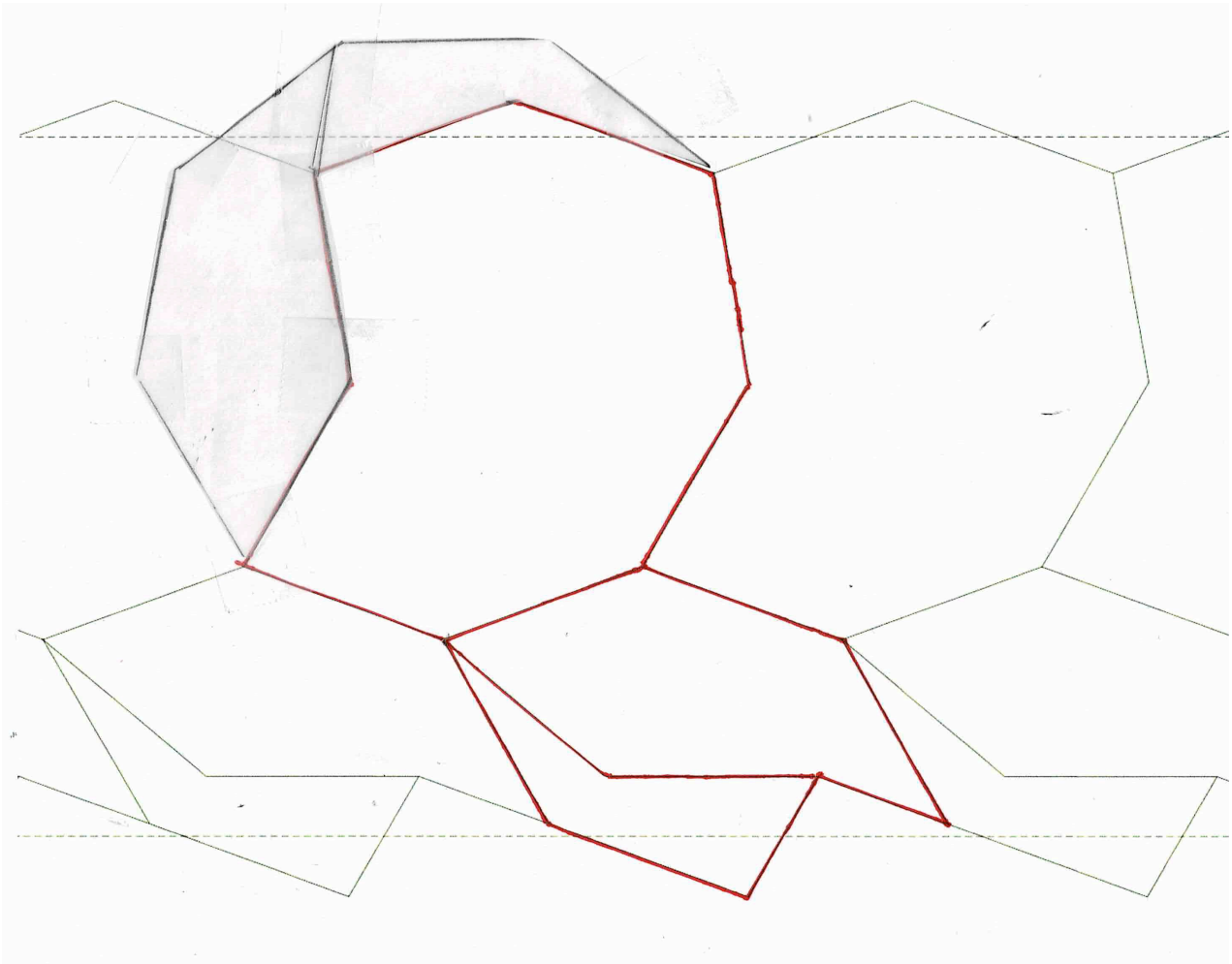
A110312

A362939

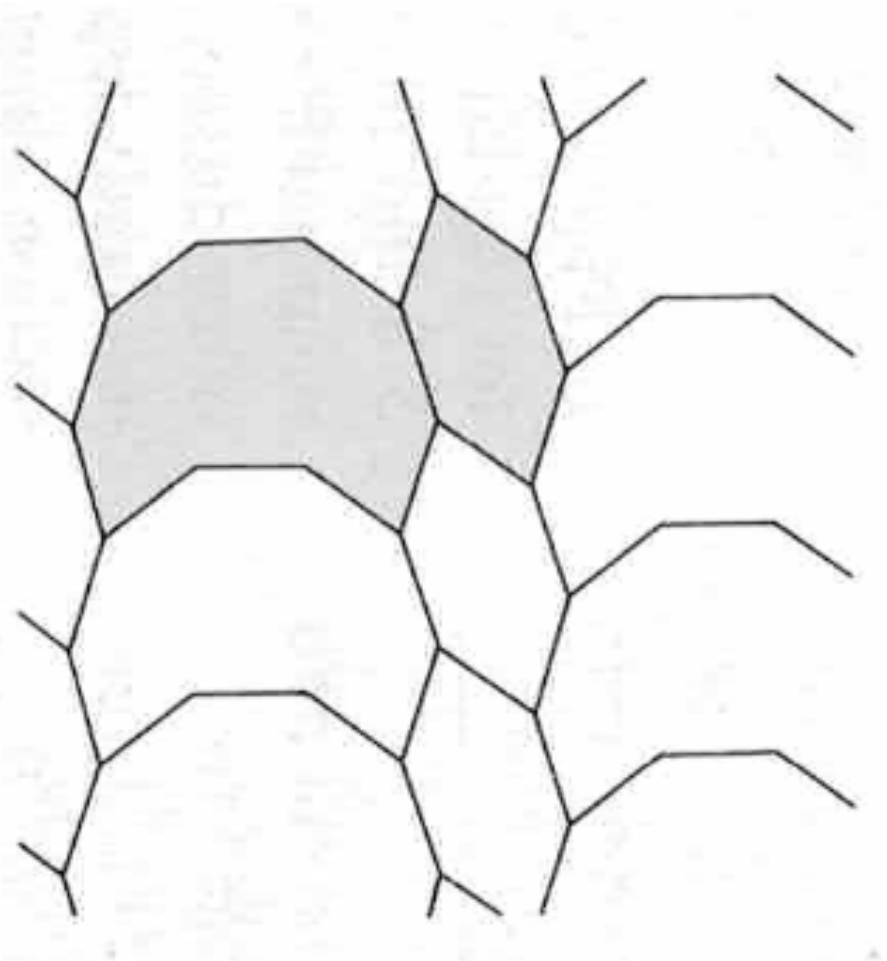
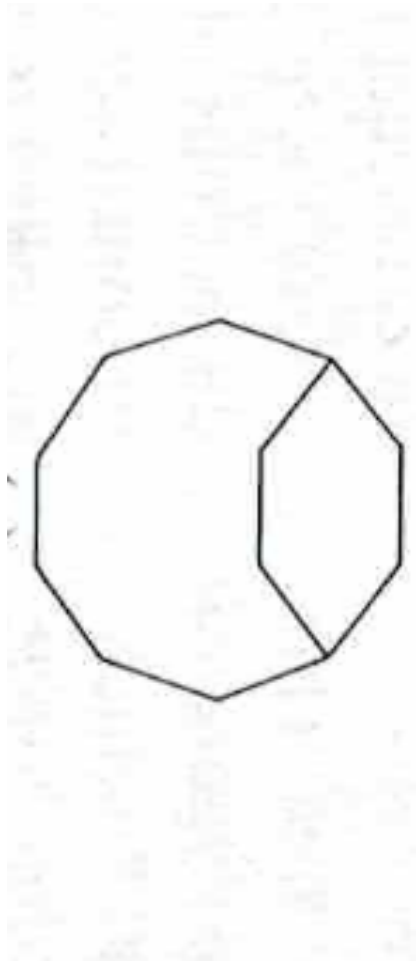
A362938



$q(5) = 2$

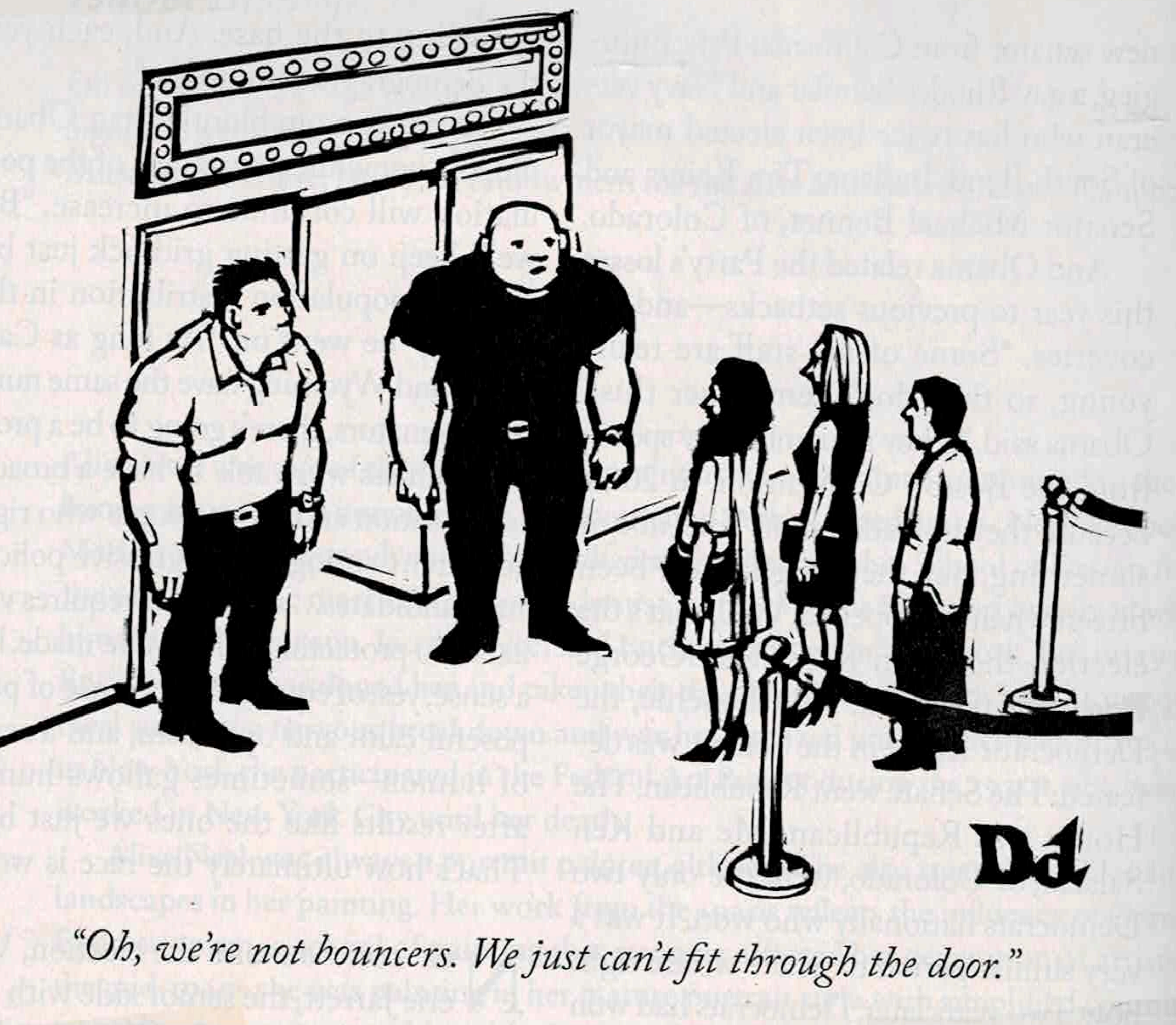


$q(9) \leq 3$ (Gavin Theobald)



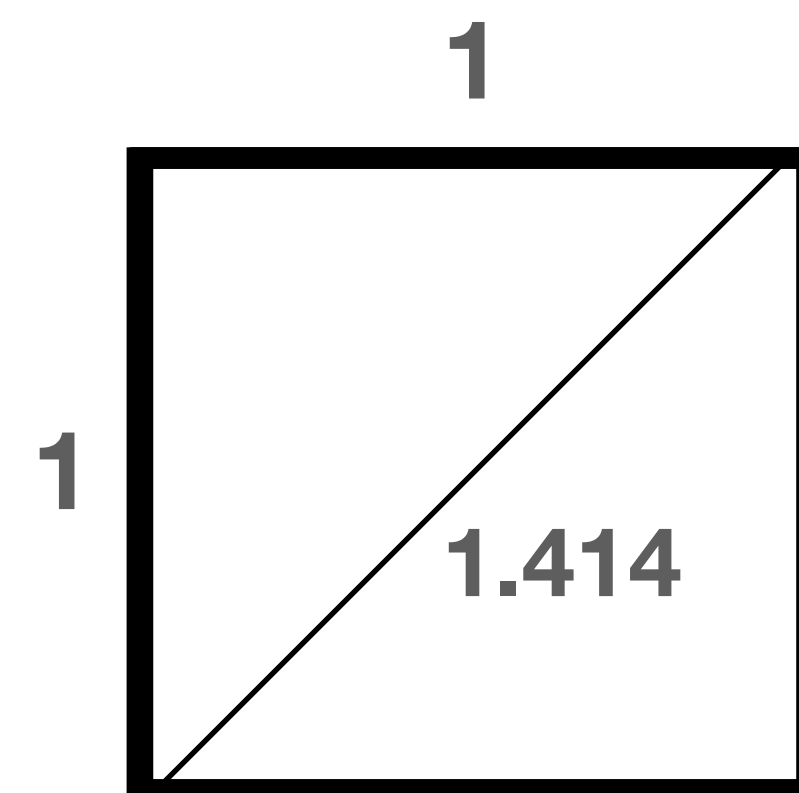
$q(10) = 2$

Dissecting polygons (cont.)

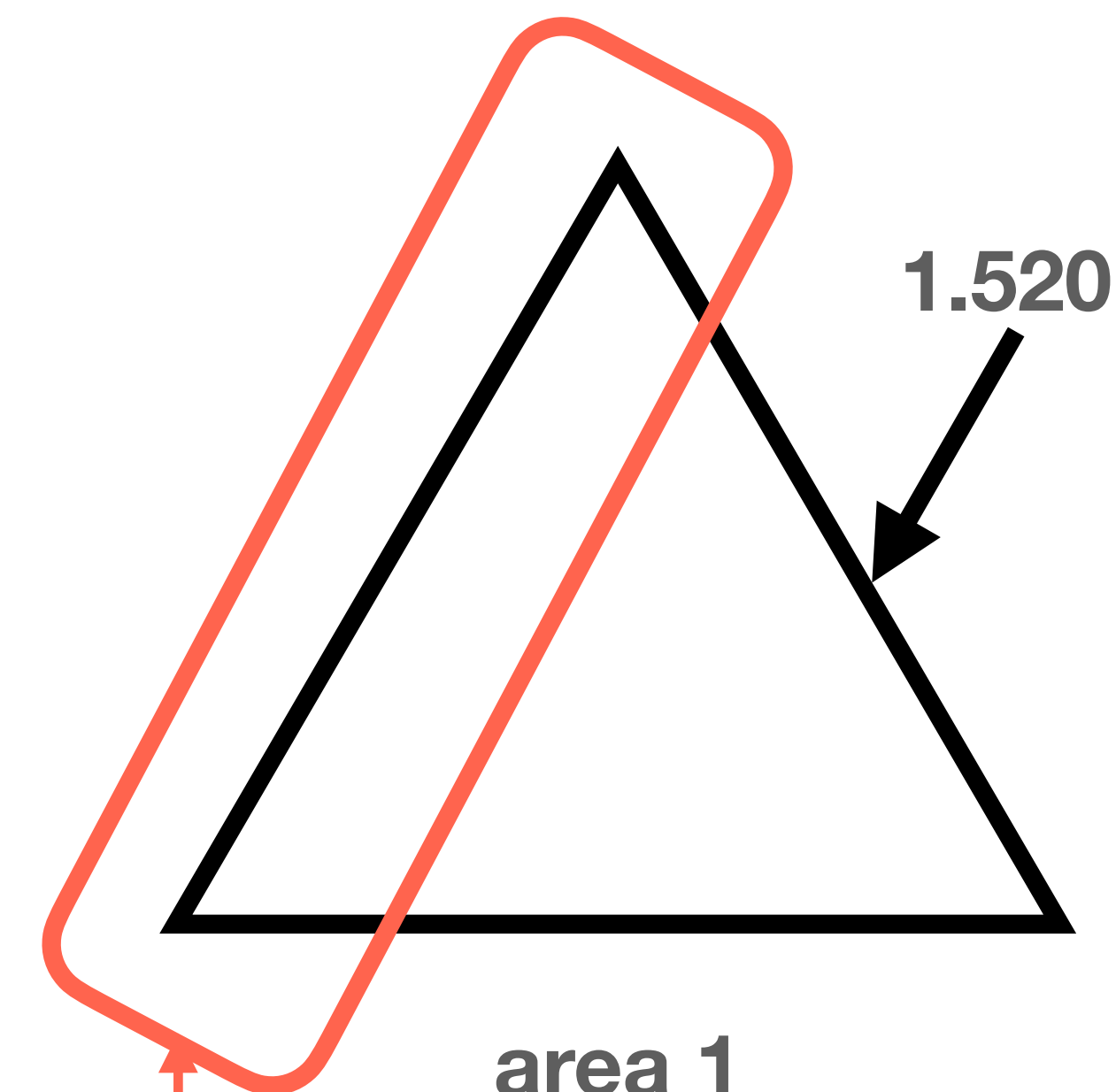


Oh, we're not bouncers. We just can't fit through the door.

Theorem $s(3) > 2$



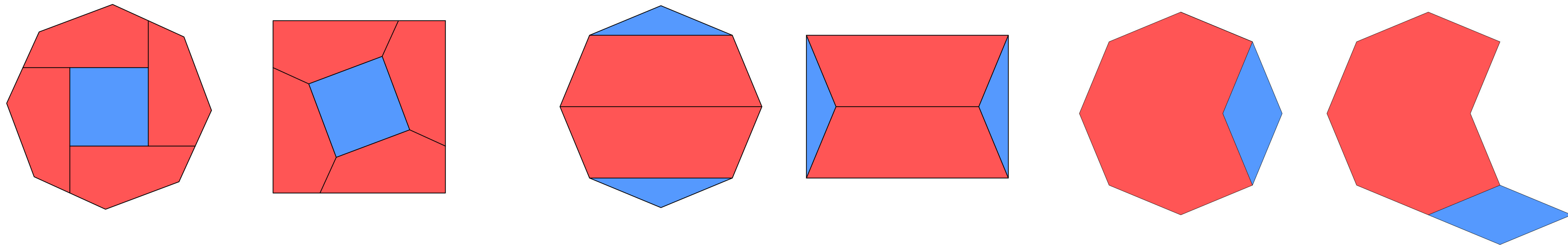
"door"



Each piece can contain at most one vertex, so at least 3 pieces!

Dissecting polygons (cont.)

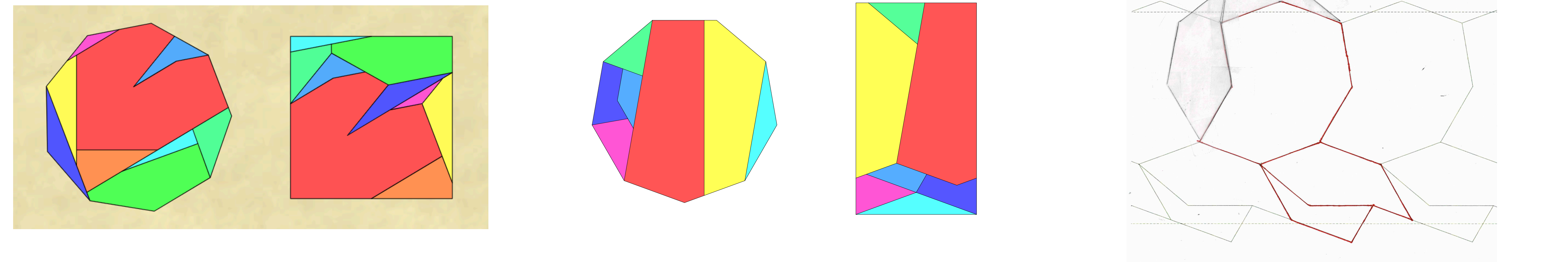
From an octagon you can get a square in 5 pieces, a rectangle in 4 pieces, and a monotile in 2 pieces:



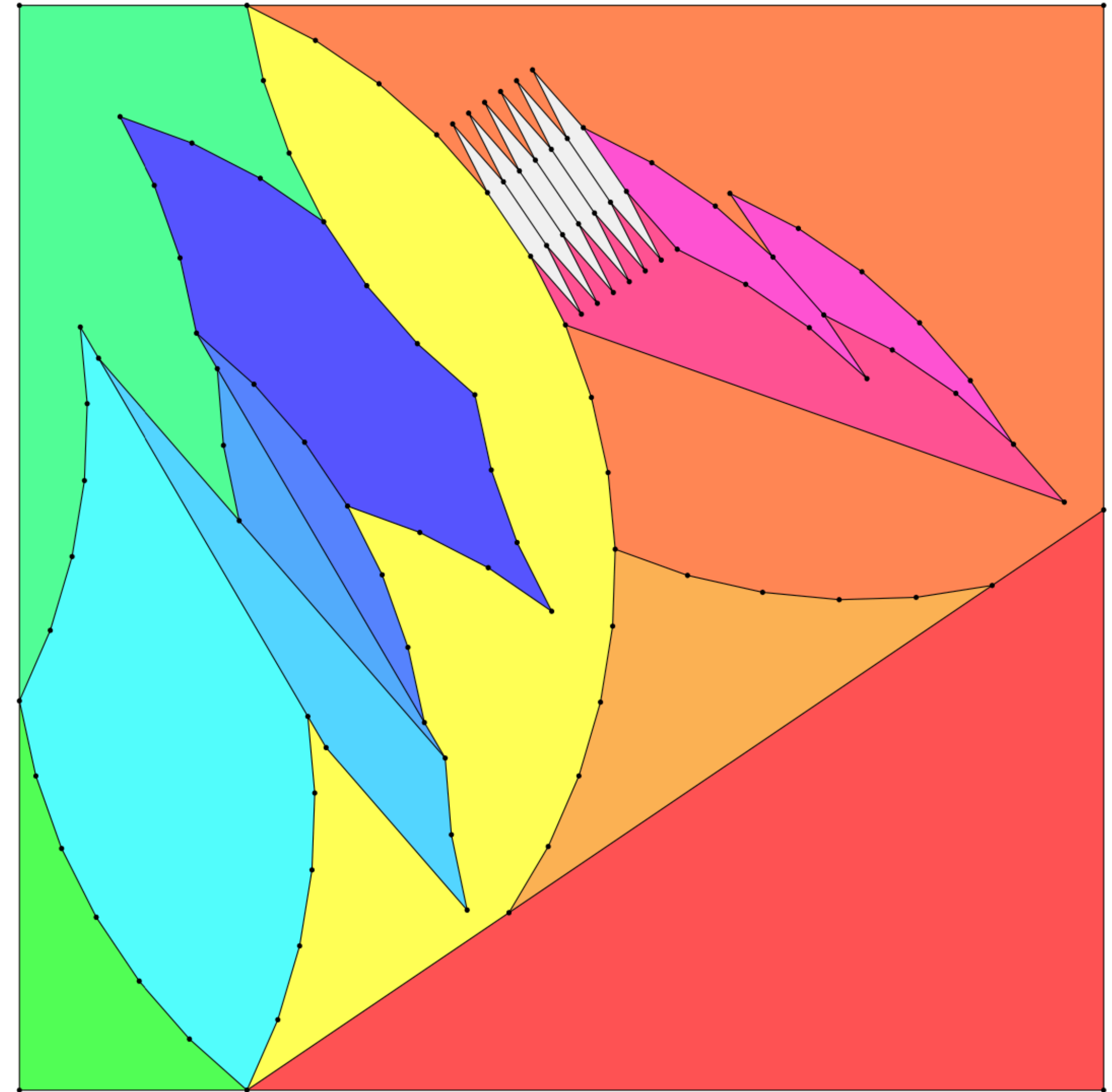
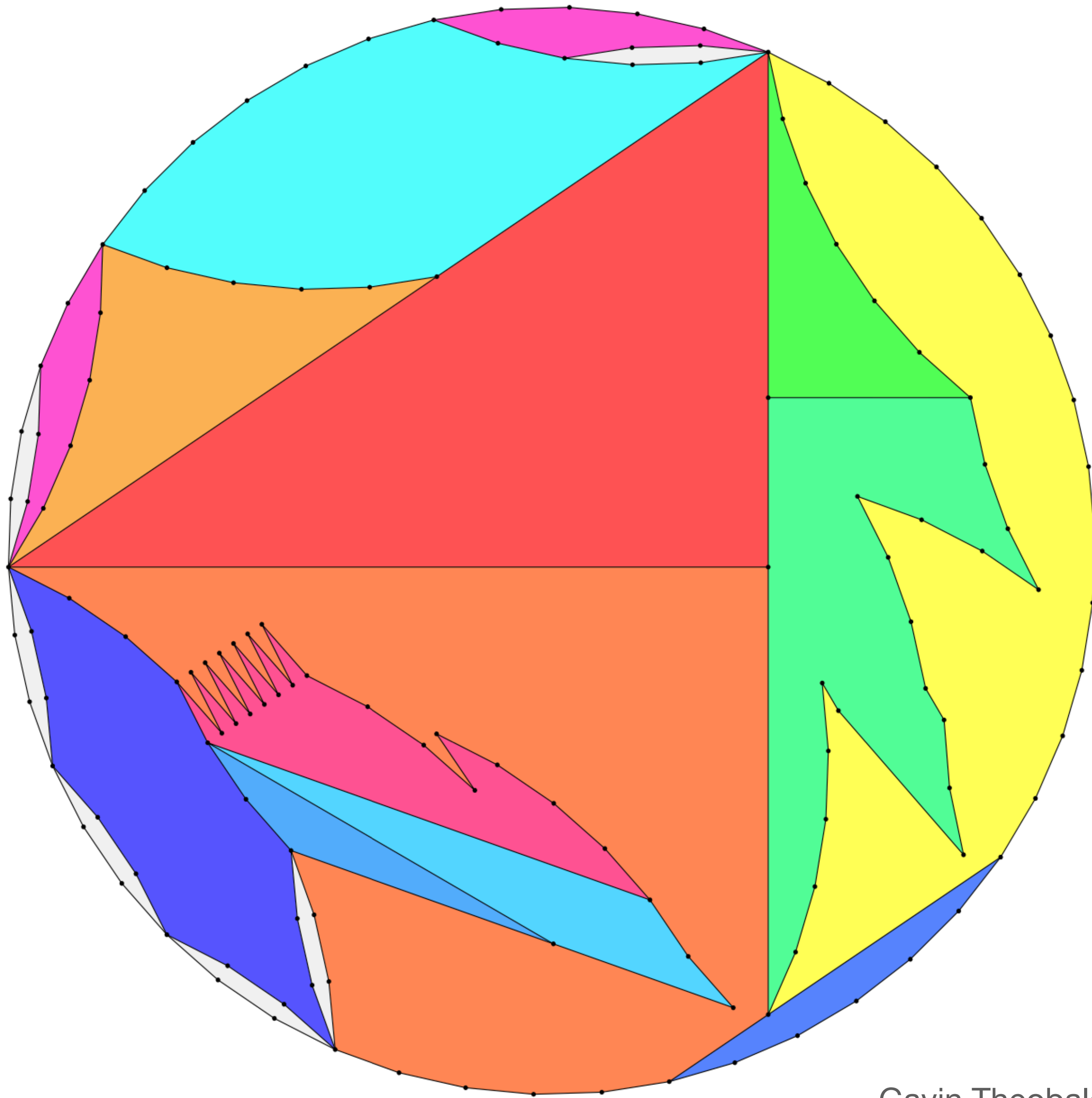
Octagon — Monotile

(2 pieces)

A 9-gon is much harder (9, 7, and 3 pieces):



Dissecting polygons (cont.)



Gavin Theobald's incredible dissection of a 50-sided polygon into 20 pieces that can be rearranged to form a square.

Goodstein Sequences

The "longest" finite sequences in the OEIS.

Of course this is meaningless, but the construction is remarkable.

Defn Hereditary base n (Ordinary) base 3: $100 = 3^4 + 2 \cdot 3^2 + 1$
 Hereditary base 3: $100 = 3^{3+1} + 2 \cdot 3^2 + 1$
 Base 2: $40 = 2^5 + 2^3$. Hereditary base 2: $40 = 2^{2+1} + 2^{2+1}$
 (No "digits" bigger than the base)

Goodstein's "bumping up" operation In base b :

In hereditary base b , replace every b by $b+1$; then subtract 1.

Base 3: $100 = 3^{3+1} + 2 \cdot 3^2 + 1 \rightarrow 4^{4+1} + 2 \cdot 4^2 + 1 - 1 = 1056$

Base 2: $40 = 2^{2+1} + 2^{2+1} \rightarrow 3^{3+1} + 3^{3+1} - 1 = 3^{28} + 3^4 - 1$
 $= 22876792455041$

Goodstein's Theorem (1944) Start with hered. rep. of n in base 2.
 Bump up to base 3, 4, 5, ... Will eventually reach 0.

$G(n) :=$ number of steps to reach 0.

Goodstein Sequences (cont.)

The Goodstein trajectory of 4 : $G(4)$:

Step 0 : $4 = 2^2$

1: $\rightarrow 3^3 - 1 = 26 = 2 \cdot 3^2 + 2 \cdot 3 + 2$

2: $\rightarrow 2 \cdot 4^2 + 2 \cdot 4 + 2 - 1 = 2 \cdot 4^2 + 2 \cdot 4 + 1 = 41$

3: $\rightarrow 2 \cdot 5^2 + 2 \cdot 5 + \cancel{2} - \cancel{1} = 60$

4: $\rightarrow 2 \cdot 6^2 + 2 \cdot 6 - 1 = 2 \cdot 6^2 + 6 + 5 = 83$

5: $\rightarrow 2 \cdot 7^2 + 7 + 4 = 109$

6: $\rightarrow 2 \cdot 8^2 + 8 + 3 = 139$

$\rightarrow \dots$

4, 26, 41, 60, 83, 109, 139, ... (A056193)

Reaches 0 at term

$$G(4) = 3 \cdot 2^{3 \cdot 2^{27} + 27} - 3 \approx 10^{121210694}$$

$G(n)$

0	1	2	3	4	5
0	1	3	5	$\approx 10^{121210694}$	$\dots$

Not in $OEIS$!

$n = 9$:

$$2^{2^{2^{70}}} + \dots \approx 10^{10^{10^{20}}}$$

Goodstein Sequences (cont.)

Triangle A399175: Row n gives Goodstein trajectory of n .

[illegible]

Riesel Numbers

The shortest sequence (one term) in the OEIS!

A076337

509203

Riesel numbers:

Odd k such that $k \times 2^i - 1$ is composite for all $i > 0$.

Keywords: nonn,bref,hard,more

Riesel numbers (cont.)

The Riesel problem: Double and add 1, repeat: how long to reach a prime?

4 - 9 - 19 - 39 - 79 - ..., or equivalently $5 \times 2^i - 1, i \geq 0$.

Double and add 1					Sequence	A052333 First prime	A050412 Steps
1	3	7	15	...	$2 \cdot 2^i - 1$	3	1
2	5	11	23	...	$3 \cdot 2^i - 1$	5	1
3	7	15	31	...	$4 \cdot 2^i - 1$	7	1
4	9	19	39	...	$5 \cdot 2^i - 1$	19	2
...	...	...	...	...	...	...	...
28	57	115	231	463	$29 \cdot 2^i - 1$	463	4
...	...	...	...	...	...	...	...
33748	67497	...	$\approx 10^{674}$	...	$33749 \cdot 2^i - 1$	$\approx 10^{674}$	2224
...	...	...	...	...	...	...	...
104916	209833	...	??	...	$104917 \cdot 2^i - 1$	$\approx 10^{102409}$	340181

See below!

But do we always reach a prime?

Riesel numbers (cont.)

We ran into this problem in 2024, while analysing an innocent-sounding sequence, [A374965](#) - see my talk in this seminar, "[A Nasty Surprise in a Sequence ...](#)", Exper. Math. Seminar, Oct 10, 2024.

The analysis stalled because we needed to find the first prime you reach when starting at 104916 and repeatedly doubling and adding one. It takes 340181 steps to reach a prime (the prime is about 10^{102409}). That was the "nasty surprise"!

But do we always reach a prime? No!

This whole subject was started by the article:

Hans Riesel, "Some Large Prime Numbers" [Swedish], Elementa 39 (1956), 258-260 (*).

(Lars Blomberg kindly provided an English translation. See [A076337](#))

Theorem (Riesel): There are infinitely many positive integers k such that the sequence $k \times 2^i - 1$, $i > 0$, contains no primes.

For a proof, see the Appendix to my 2024 talk (the last two slides).

The smallest example he found was $k = 509203$.

No one has found a smaller example.

Since 1998 there has been a vast project underway to prove that 509203 is indeed the smallest example.

To prove it, for each of the 254601 odd numbers $k < 509203$, the project tries to find a prime in the sequence $k \times 2^i - 1$, $i > 0$.

This has now been done for all except 42 values of k . See:

Ray Ballinger and Wilfrid Keller, [The Riesel Problem: Definition and Status](http://www.prothsearch.com/rieselprob.html) [http://www.prothsearch.com/rieselprob.html].

1973

Betelgeuse - First contact:

To demonstrate that there is intelligent life here, send the following six sequences (omitting of course the OEIS A-numbers)!

- 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, ... ([A000027](#))
- 2, 3, 5, 7, 11, 13, 17, 19, 23, 31, ... ([A000040](#))
- 1, 1, 2, 3, 5, 8, 13, 21, 34, 55, 89, ... ([A000045](#))
- 1, 24, 24, 96, 24, 144, 96, 192, 24, 312, ... ([A000118](#))
- 1, 240, 2160, 6720, 17520, 30240, 60480, 82560, 140400, ... ([A004009](#))
- 7920, 95040, 175560, 443520, 604800, 10200960, 44352000, ... ([A001228](#), see next slide)

[Other sequences that could be used: [A000043](#), [A046051](#). See also [A248747](#)].

[Hans Freudenthal, "Lincos: Design of a Language for Cosmic Intercourse", North-Holland, 1960].

Betelgeuse (cont.):

7920
95040
175560
443520
604800
10200960
44352000
50232960
244823040
898128000
4030387200
145926144000
448345497600
460815505920
495766656000
42305421312000
64561751654400
273030912000000
51765179004000000
90745943887872000
4089470473293004800
4157776806543360000
86775571046077562880
1255205709190661721292800
4154781481226426191177580544000000
8080174247945128758864599049617107570057543680000000000

(For the sixth sequence, which lists the 26 orders of
the sporadic simple groups, send the full list! (A001228))

Keith Numbers

A remarkable sequence with unusual and unexplained growth

Keith numbers: 14, 19, 28, 47, 61, 75, 197, 742, 1104, ... (A007629)

Defn.: A d-digit number N is a **Keith number** if the d-th order Fibonacci recurrence started with the digits of N contains N.

Examples: 14: 1, 4, | 5, 9, 14 197: 1, 9, 7, | 17, 33, 57, 107, 197

Mike Keith, 1987. As of July 2026, 157 Keith numbers are known, complete through 45 digits

- 1 14
- 2 19
- 3 28
- 4 47
- 5 61
- 6 75
- 7 197
- 8 742
- 9 1104
- 10 1537
- 11 2208
- 12 2580
- 13 3684
- 14 4788
- 15 7385
- 16 7647
- 17 7909
- 18 31331
- 19 34285
- 20 34348

1987

Keith Numbers (cont.)

Sequence extended by:

Daniel Lichtblau,

"Jesterjunk" (github),

and others

1 14

2 19

3 28

4 47

5 61

6 75

7 197

8 742

9 1104

10 1537

11 2208

12 2580

13 3684

14 4788

15 7385

16 7647

17 7909

18 31331

19 34285

20 34348

... ..

138	215697830679154524503635806813270373461
139	1004566042648580249092683926888439949414
140	1816340802304828405869941580057044476938
141	3667665486047337607150308556285662810291
142	7322328822325833732474753985294810666035
143	8065458946484467940332456027867108839048
144	8425728713644186076789822654950556667780
145	15424828410226613515507443774158669599960
146	30081830087987579923672365111897261127043
147	246886024248854821622701598693112655546442
148	262352999160589366533639168718163151078418
149	275098759996873662461317925206088540716562
150	595772628889363625726896288920522288914169
151	909174340023749619572306357633062920562913
152	2112877907698905120555962354671456805629810
153	4290808821126219499179742927817326615662906
154	17467373971637404548114635268844355733328377
155	71398493431907987712771787295396434920302163
156	154692578281663954478716671357617796091461825
157	290985559683328038709143906270221495951569185

NOTES:

Keith Numbers (cont.)

1. How was it possible to search all numbers up to 10^{45} ?

Answer: Daniel Lichtblau (2006, see link in [A007629](#)) reduced problem to a set of simultaneous diophantine equations, for which there are many algorithms.

To find all 3-digit Keith numbers abc ($1 \leq a \leq 9$, etc)

$a, b, c, a+b+c, a+2b+2c, 2a+3b+4c,$
 $4a+6b+7c, 7a+11b+13c, \dots$

- coefficients are "tribonacci" (known) numbers.

Want a, b, c such that for example

$$\begin{array}{ccccccc} 7a & + & 11b & + & 13c & = & 100a + 10b + c \\ 1 & & 9 & & 7 & = & 197 \end{array}$$

and
is a solution.

Keith Numbers (cont.)

2. **A050239** gives the number of d-digit Keith numbers, for $d = 1$ to 45:

0, 6, 2, 9, 7, 10, 2, 3, 2, 0, 2, 4, 2, 3, 3, 3, 5, 3, 5, 3, 1, 1, 3, 1, 1, 3, 7, 1, 2, 5, 2, 4, 6, 3, 4, 5, 4, 9, 2, 6, 2, 5, 2, 2, 2

That is a remarkable sequence! One would expect it to either grow or end!

Open Problem:

Either show there are only finitely many Keith numbers,

or else show that for $d > 10$ a Keith number always exists!

[See also Martin Klazar and Florian Luca, "Counting Keith Numbers", J. Int. Seqs., 10 (2007)]

1994

The Most Wanted Primes

When is $1234\dots n$ prime?

$$1234567 = 127 \cdot 9721$$

$$12345678910111213 = 113 \cdot 125693 \cdot 869211457$$

John Selfridge asks if there are infinitely many such primes (Guy, UPINT 2nd Ed, 1994, p. 9).
The answer should be Yes, on probabilistic grounds.

None are known!

It is said that none exist for $n < 10^6$ - see Wolfram Mathworld, "Smarandache Prime",
last updated June 2018, quoting Serge Batalov.

Has there been no progress since 2018?

Expected number in first million terms is 0.6 - Ernst Mayer, 2013.

- See [A007908](#) for further information

The probabilistic argument: $b_n = 1234\dots n$ has about $n \log n$ digits.

Probability b_n is prime is about $1 / \log(n \log n)$,

The sum of $1 / \log(n \log n)$ diverges, so expect oo many prime b_n

Most wanted primes (cont)

A Memorable Prime

12345678910987654321

When is $123\dots n-1 \text{ } n \text{ } n-1\dots 321$ prime?

It is a square: $11\dots 1^2$ for $n \leq 9$.

Prime for $n = 10, 2446$ (Shyam Gupta). See [A359148](#).

Or, in base b , when is $123\dots b-1 \text{ } b \text{ } b-1\dots 321$ a prime?

Prime for $b = 2, 3, 4, 6, 9, 10, 16, 40, 104, 8840$

(David Broadhurst, Aug 2015, [A260343](#))

2004

The Curling Number Conjecture

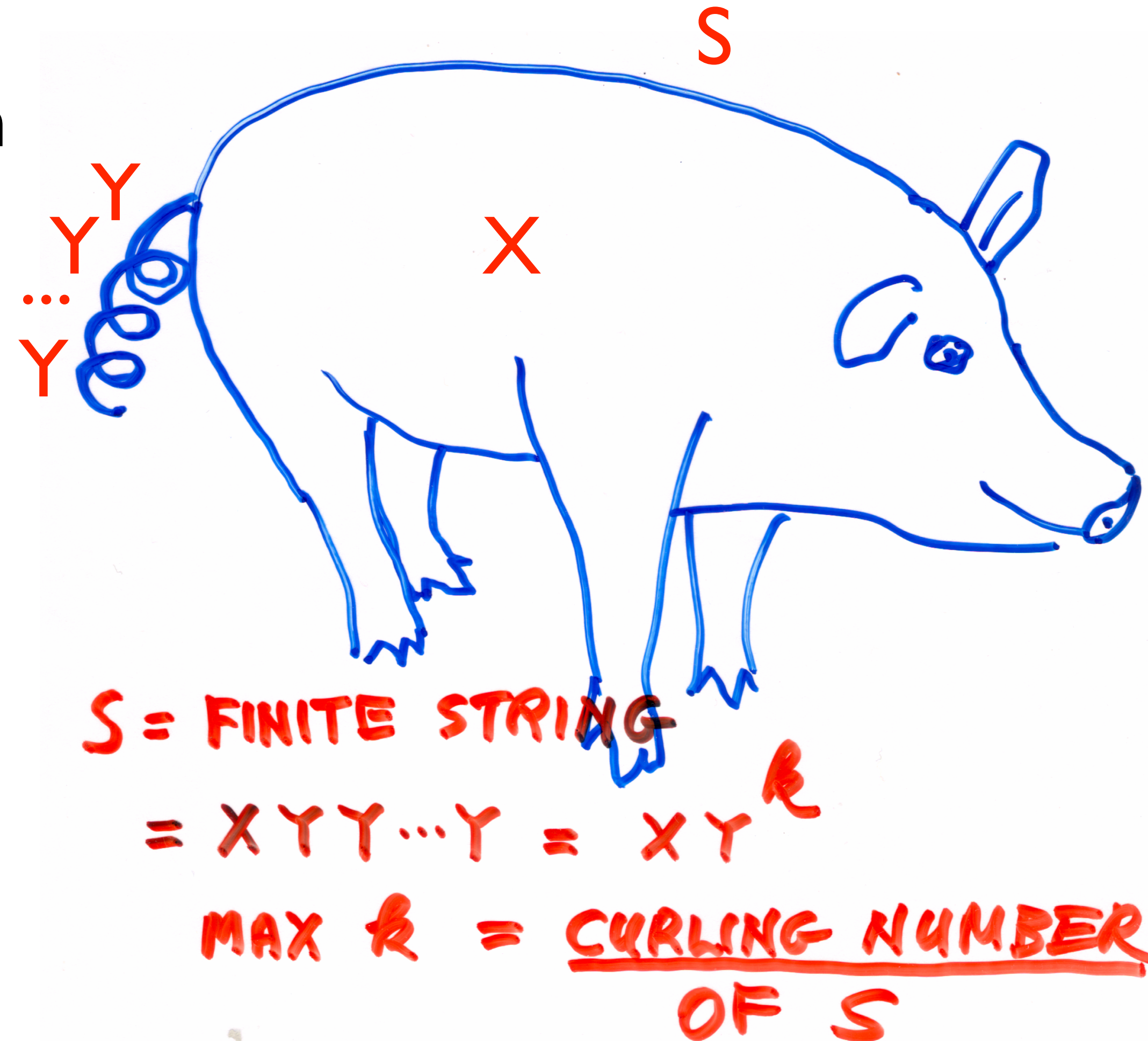
Could AI help here?

[See [A216955](#), [A181935](#), [A116909](#)]

B. Chaffin et al., On the curling number of integer sequences, J. Integer Sequences, 16 (2013), #13.4.3; arXiv:1212.6102.

The Curling Number Conjecture (cont.)

Definition of Curling Number



$$S = 7522522522, \quad k = 3$$

CURLING NUMBER CONJECTURE

- START WITH ANY FINITE STRING
- APPENDS CURLING NUMBER
- REPEAT
- THEN MUST REACH A 1 !?

E.G.

START : 2 2 2 3 2 2

THEN

2 3 2 2 2 3 3 2 1 ...

↑
BOO!

Problem: Prove or disprove
the Curling Number Conjecture

2004

Gijswijt's Sequence

Dion Gijswijt, 2004

A090822

**Fokko v. d. Bult, Dion Gijswijt, John Linderman,
N. J. A. Sloane, Allan Wilks ([J. Integer Seqs.](#), 2007)**

Gijswijt's Sequence G (cont.)

Start with 1, always append curling number

1 1 2
 1 1 2 2 2 3
 1 1 2
 1 1 2 2 2 3 2
 1 1 2
 1 1 2 2 2 3
 1 1 2
 1 1 2 2 2 3 2 2 2 3 2 2 2 3 3 2
 1 1 2

$$a(220) = 4$$

G	curling number
1	1
1 1	2
1 1 2	1
1 1 2 1	1
1 1 2 1 1	2
1 1 2 1 1 2	2!
1 1 2 1 1 2 2	2
1 1 2 1 1 2 2 2	3!

(A090822)

Is there a 5 in G?

300,000 terms: no 5

$2 \cdot 10^6$ terms: no 5

10^{120} terms: no 5

NJAS, FvdB: first 5 at about term $10^{10^{23}}$

Gijswijt, continued

First n appears at about term

$n - 1$
...
5
4
3
2
2

Conjectured by Fokko v. d. Bult, Dion Gijswijt, John Linderman,
N. J. A. Sloane, Allan Wilks ([J. Integer Seqs.](#), 2007).

Proved by Levi van de Pol, The Growth Rate of Gijswijt's Sequence,
[J. Int. Seq.](#), (2025) Vol. 28, Art. No. 25.4.6.

Gijswijt, continued

NOTE! The Curling Number Conjecture would imply that Gijswijt's sequence G is universal: every starting sequence S not containing a 1 will eventually merge with G , that is, it will eventually become STG for some sequence T

Gijswijt, continued

Proofs could be simplified if Curling Number
Conjecture were true

How far can you get with an initial
string of n 2's and 3's
(before a 1 appears)?
Call this $\mu(n)$ (or mu(n))
(the number of added terms before hitting a 1)

(A217208)

What's the best start with n 2's and 3's?

n=4: best start is S = 2323 which is followed T = 2223 then 1, so $\mu(4) = 4$

**n=8: best start is S = 23222323 followed by T of length 58
2223222322232223232223222322232223232223222322332
then 1, so $\mu(8) = 58$**

**n=22: best start is S = 2322322323222323223223 followed by
T of length 120 then 1, so $\mu(22) = 120$**

0, 2, 2, 4, 4, 8, 8, 58, 59, 60, 112, 112, 112, 118, 118, 118, 118
118, 119, 119, 119, 120, 120, 120, 120, 120, 120, 120, 120, 120,
120, 120, 120, 120, 120, 120, 120, 120, 120, 120, 120, 120,
120, 120, 120, 120, 131, 131, 131, 131, 131, 131, 131, 131,
131, 131, 131, 131, 131, 131, 131, 131, 131, 131, 131, 132, 132,
132, 132, 132, 132, 132, 132, 133, 173, 173, 173, 173

$\mu(n)$ for $n \geq 1$
(A217208)

We have exact values of $\mu(n)$ for $n \leq 48$,
and strong conjectures for $n \leq 80$ (Ben Chaffin)

**What is $\mu(n)$ for $n > 48$?
How does it grow?**

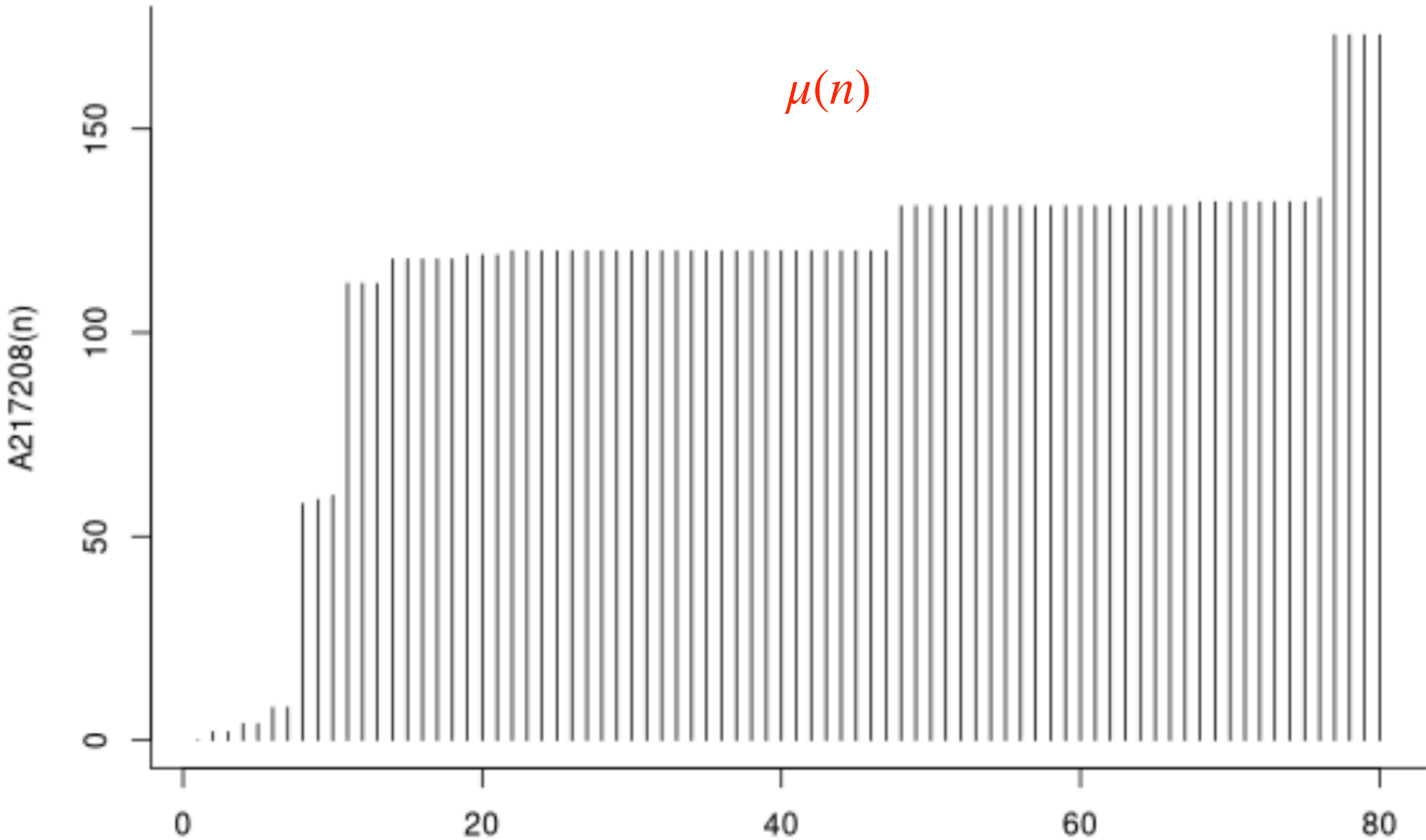
**What are these exceptional
starting sequence?**

**Find how to construct and
generalize them!**

**Conjecture: It is a good idea
to assume that 33 does not
appear in the starting sequence,
which should therefore
consist of short runs of 2's
separated by single 3's.**

(A217208)

Pin plot of A217208(n)



2013

The Fouriest Transform of n

IT'S CALLED A FOURIER TRANSFORM WHEN YOU TAKE A NUMBER AND CONVERT IT TO THE BASE SYSTEM WHERE IT WILL HAVE MORE FOURS, THUS MAKING IT "FOURIER." IF YOU PICK THE BASE WITH THE MOST FOURS, THE NUMBER IS SAID TO BE "FOURIEST."

$624_{10} \rightarrow 1160_8$
 $\downarrow$
 440_{12}
 $\downarrow$
 $4444_5 \checkmark$

Teaching math was way more fun after tenure.

Zach Weinersmith, Saturday Morning Breakfast Cereal, Feb 1, 2013

39 in base	2		1	0	0	1	1	1
	3				1	1	1	0
	4					2	1	3
	5					1	2	(4)
	6					1	0	3
	7						5	(4)
	...							

124 is Four-ier than 213!

In fact 124 is the (smallest) Four-iest transfom of 39

so $a(39) = 124$

In general, Write n in that base b

≥ 4

where you get the most 4's

$a(10)=14$ (use base 6)

0, 1, 2, 3, 4, 11, 12, 13, 20, 14, 14, 14, 14, 14, 24, 14, 24,...

A268236

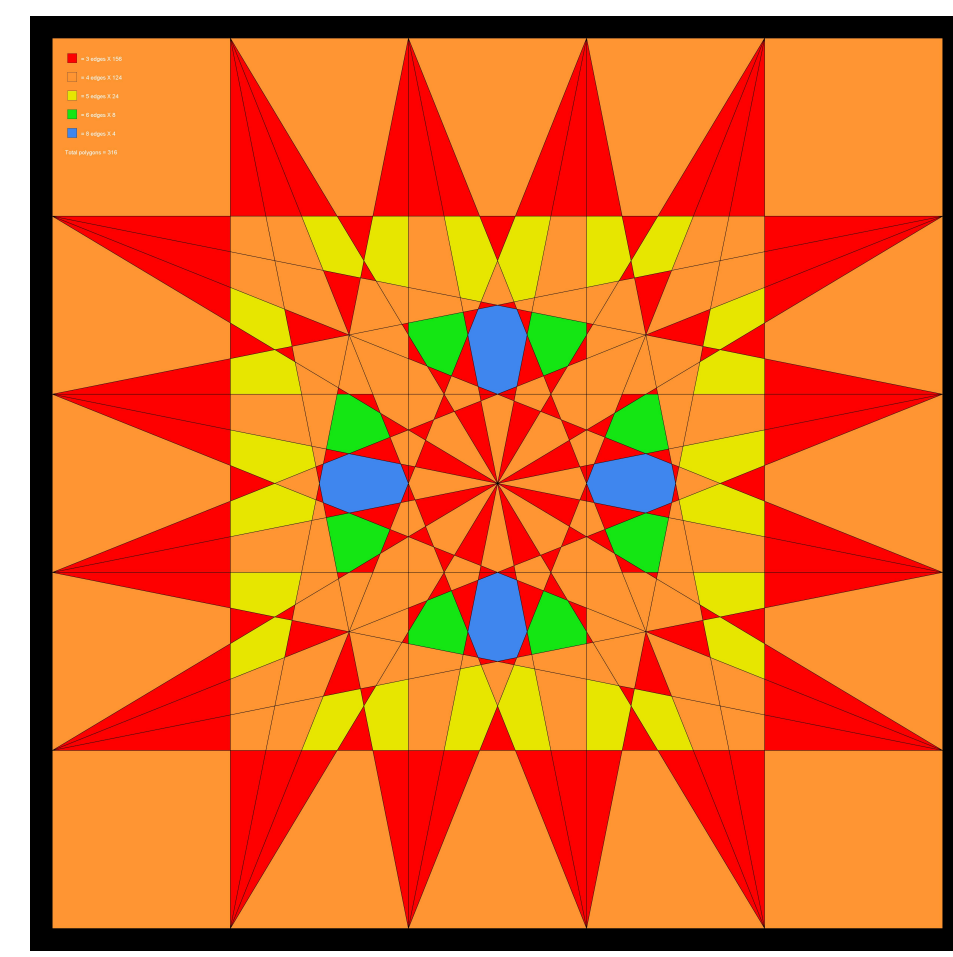
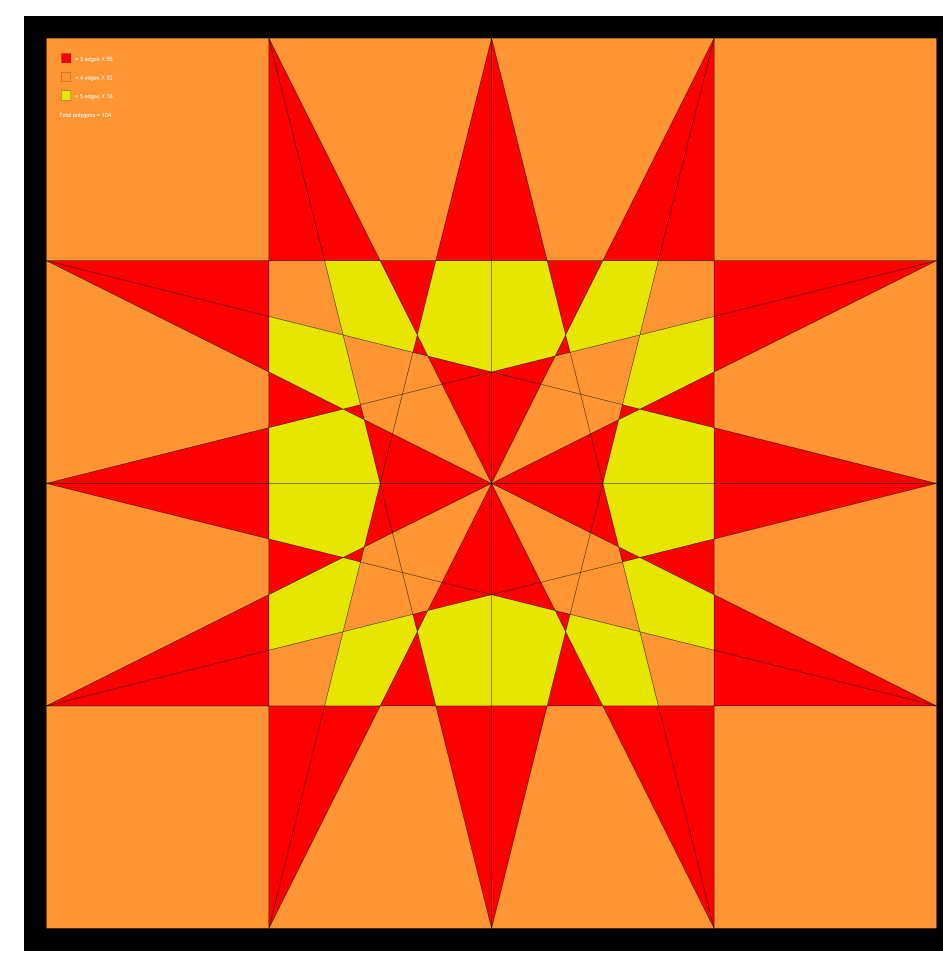
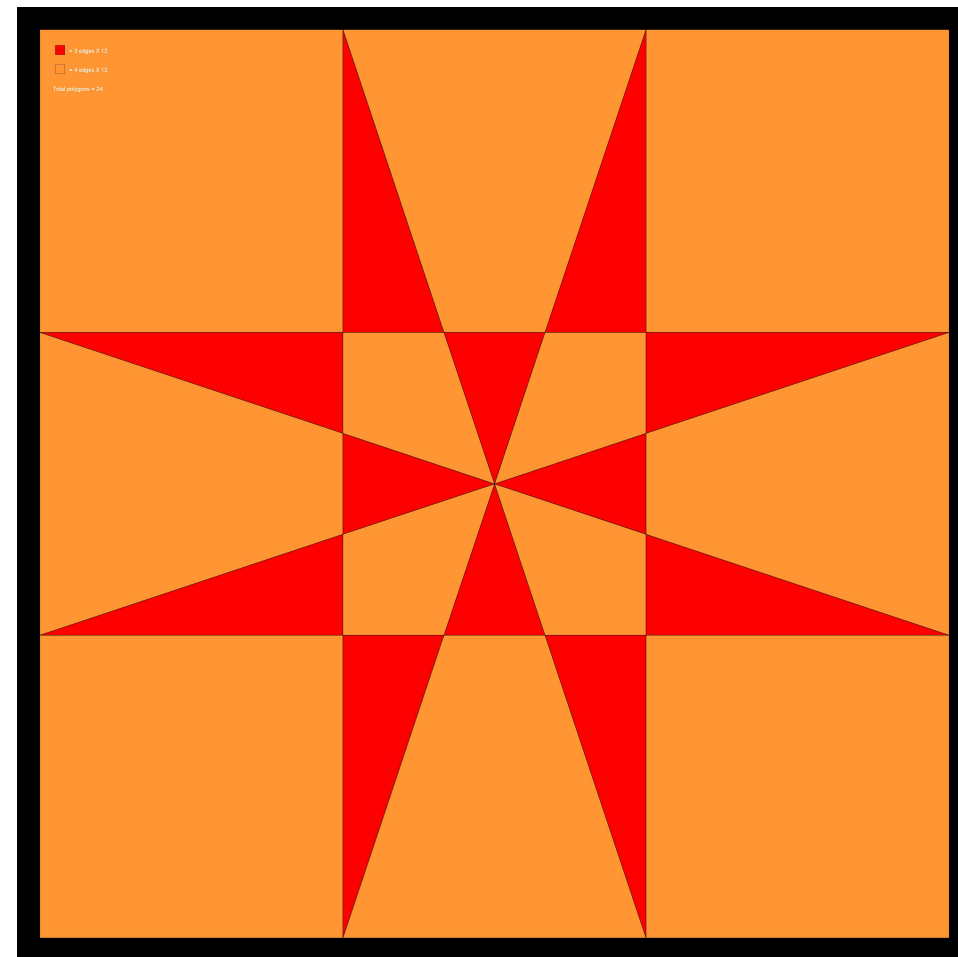
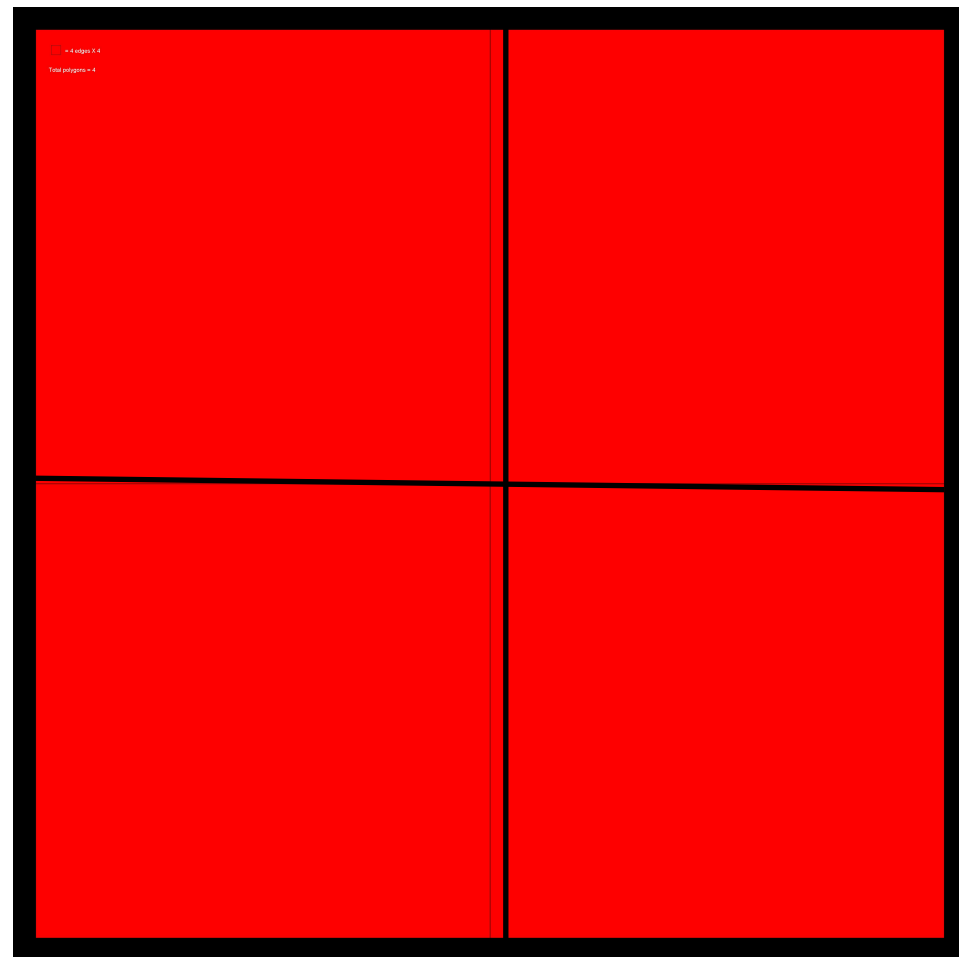
2022

Best rug design, and a sequence needing a formula

Scott Shannon's Sequence **A355798**

Place $n-1$ points on each side of a square,
join each point to every point on the opposite side.
How many regions?

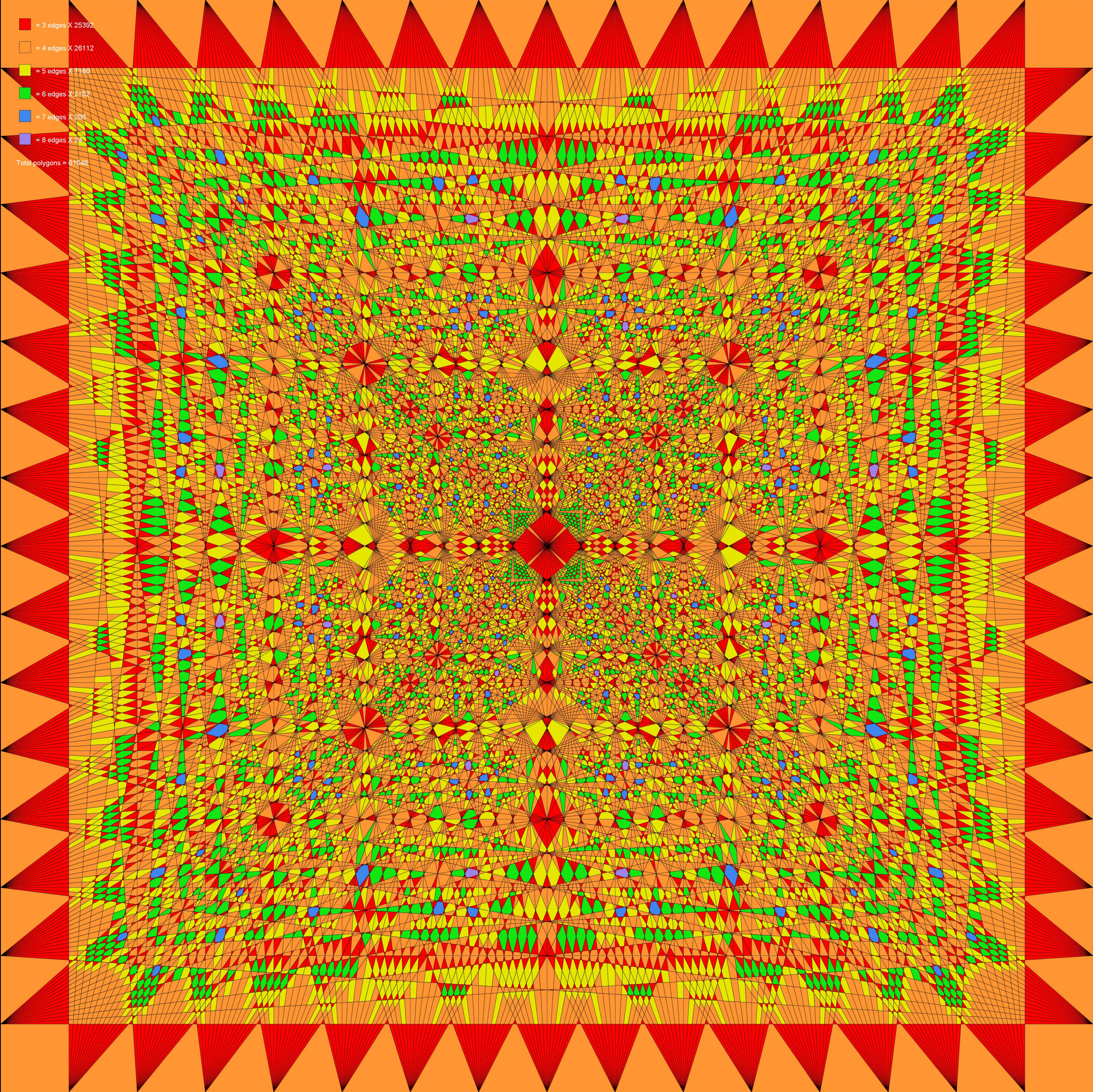
1, 4, 24, 104, 316, 712, 1588, 2816, 4940, 7672, ...



Open Problem: Have 40 terms, need a formula

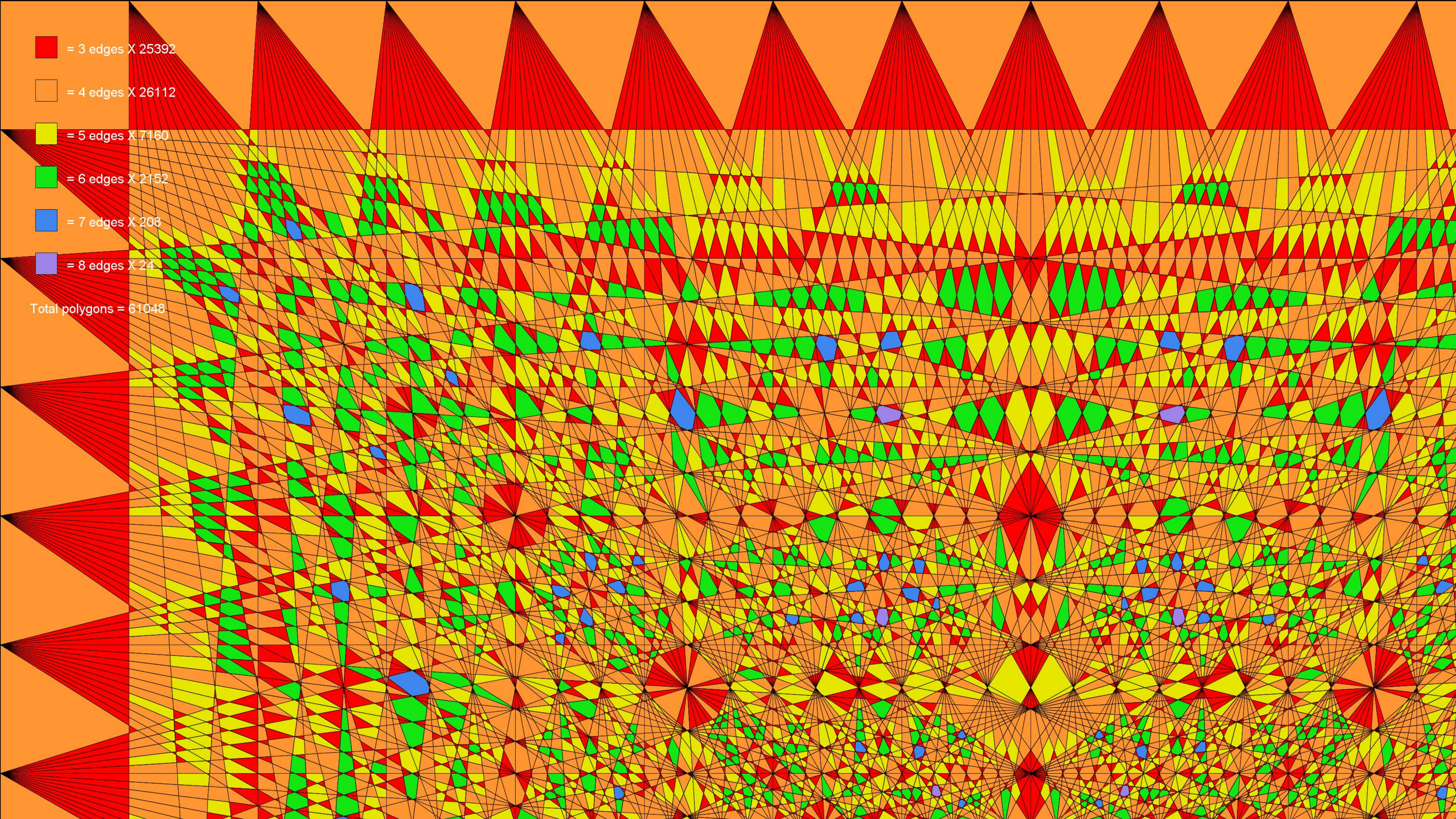
Also A355799 (vertices) and A355800 (edges)

Scott Shannon's Magic Carpet



A355798

n=16
61408 regions



Other sequences with many terms, need formulas

2

From the analysis of LES [A090252](#):

3, 1, 15, 33, 61, 97, 121, 129, 133, 245, 265, 389, 481, 485, 489, 2065, ...

<https://njas.blog/2022/06/03/the-two-up-sequence-a090252/>

3

Simple interior nodes in 1 X n grid, [A334701](#):

1, 6, 24, 54, 124, 214, 382, 598, 950, 1334, ...

Have 500 terms

[Solved by Jonas Karlsson]

4

Also [A067151](#), [A349784](#), [A350606](#).

5

Look for words “It would be nice to have a formula ...” in OEIS entries.

2023

The Scariest Sequence in the OEIS

2023 The scariest
sequence in the OEIS:
A360521 (a million terms)

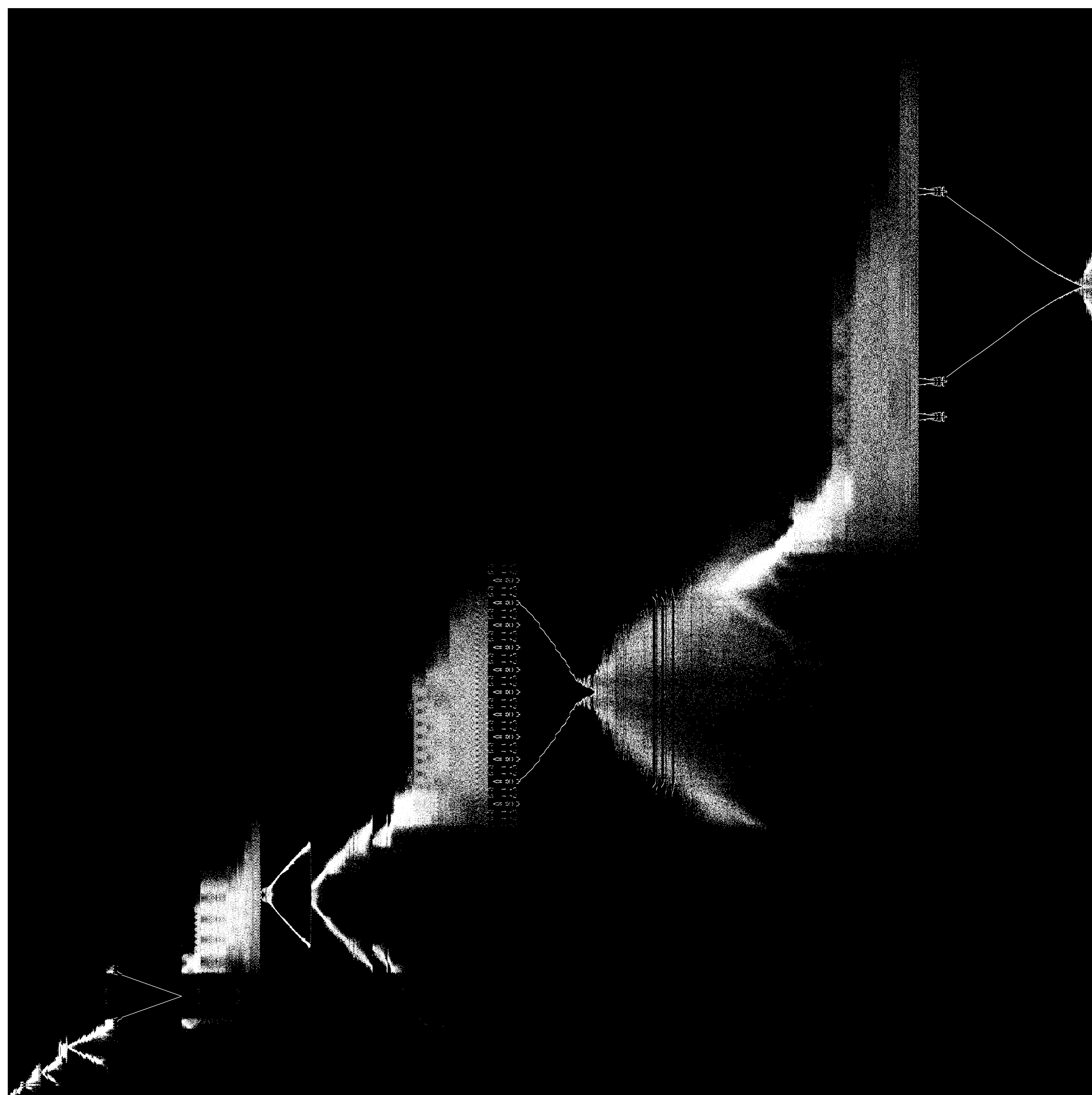
It is worth going to [A360521](#) and
looking at all the illustrations!

The sequence and its images are
the work of Scott R. Shannon in
2023.

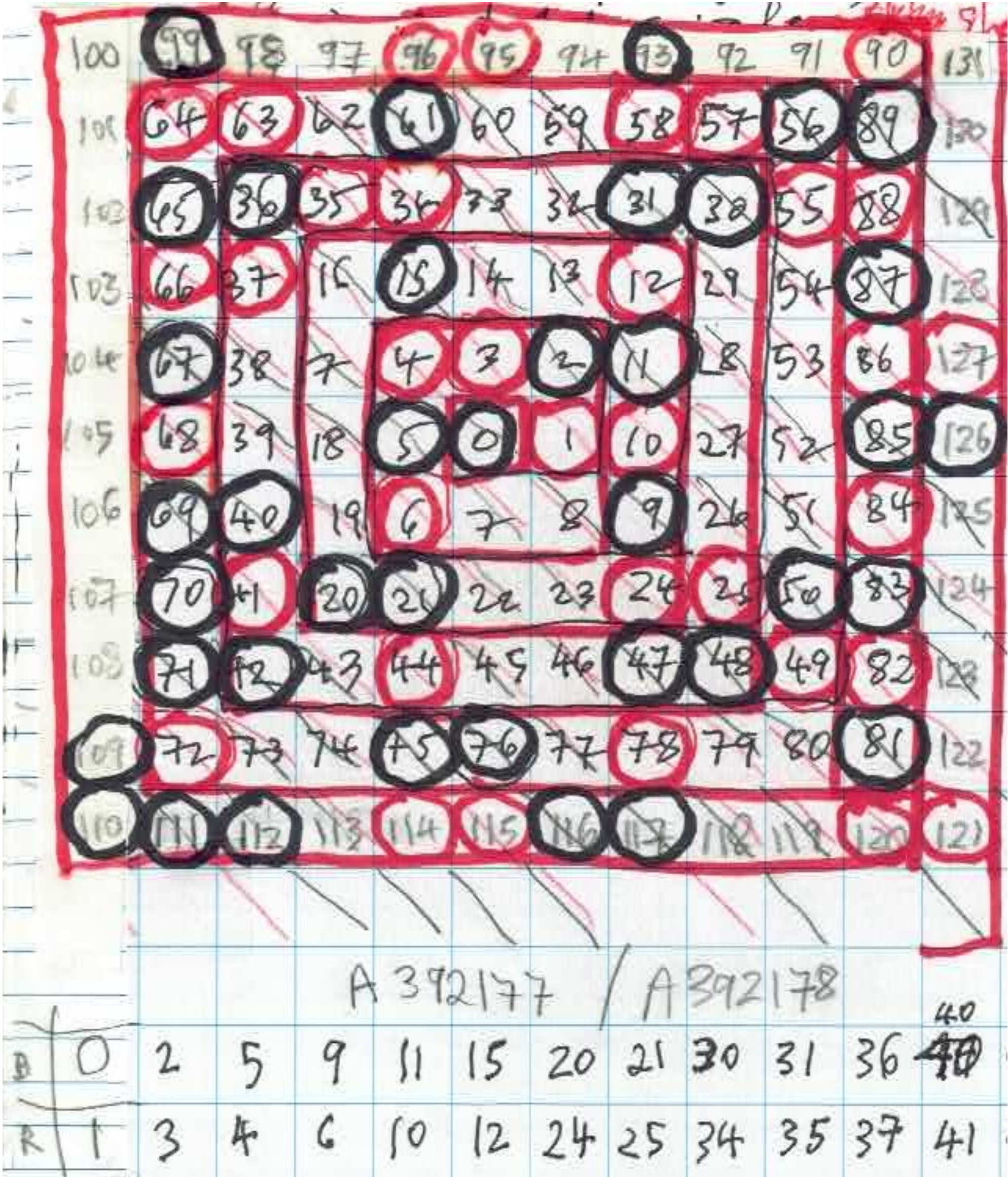
The composer Gustavo Díaz-
Jerez's work Fuzzball

is based on this sequence (and dedicated to the OEIS).

There is a performance by the composer
on [Youtube](#). (turn up volume to hear it.)



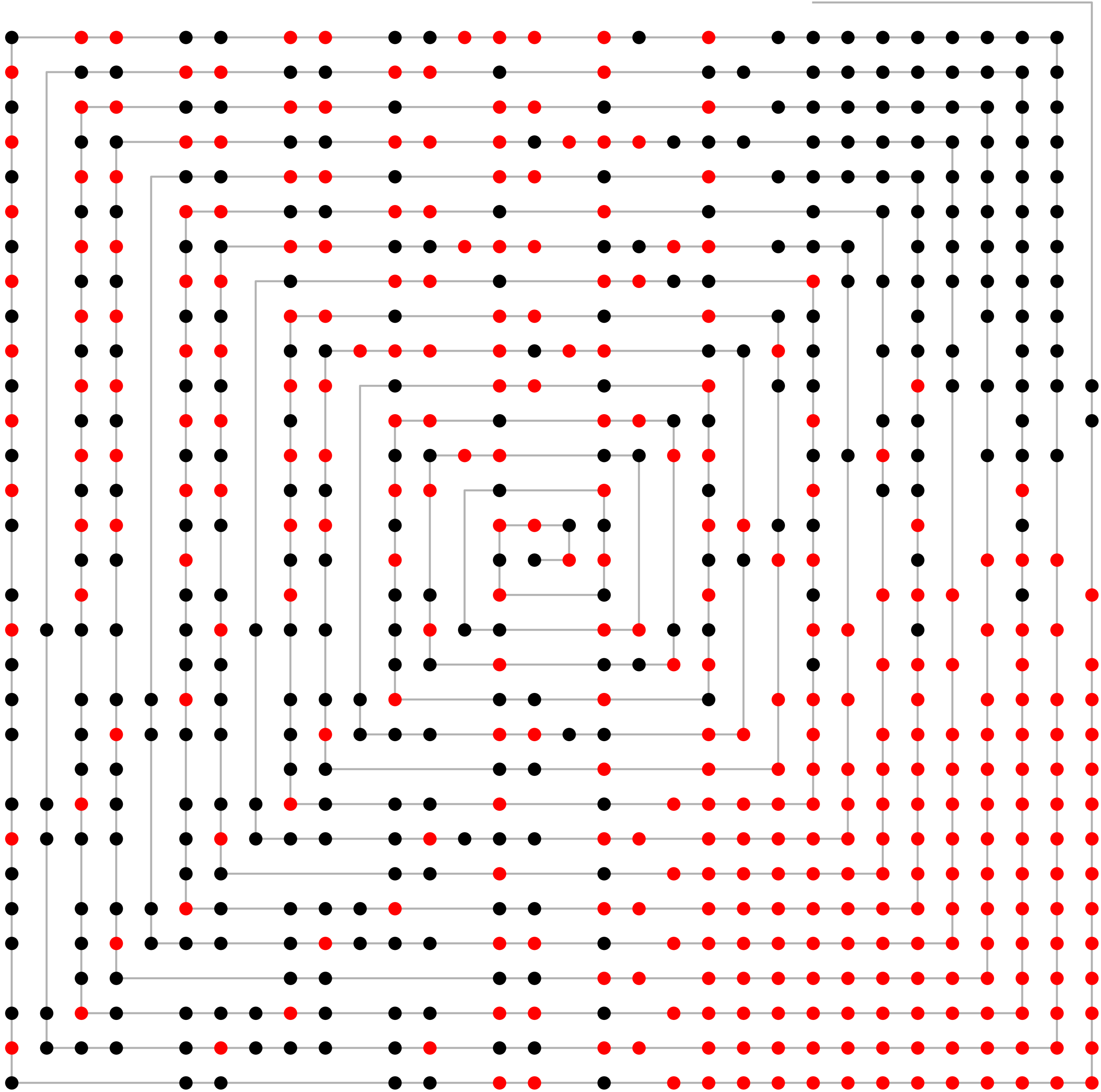
The Red and the Black Knights



A discovery of Jonas Karlsson, analysed by Michael Branicky, J.K., and N.J.A.S.

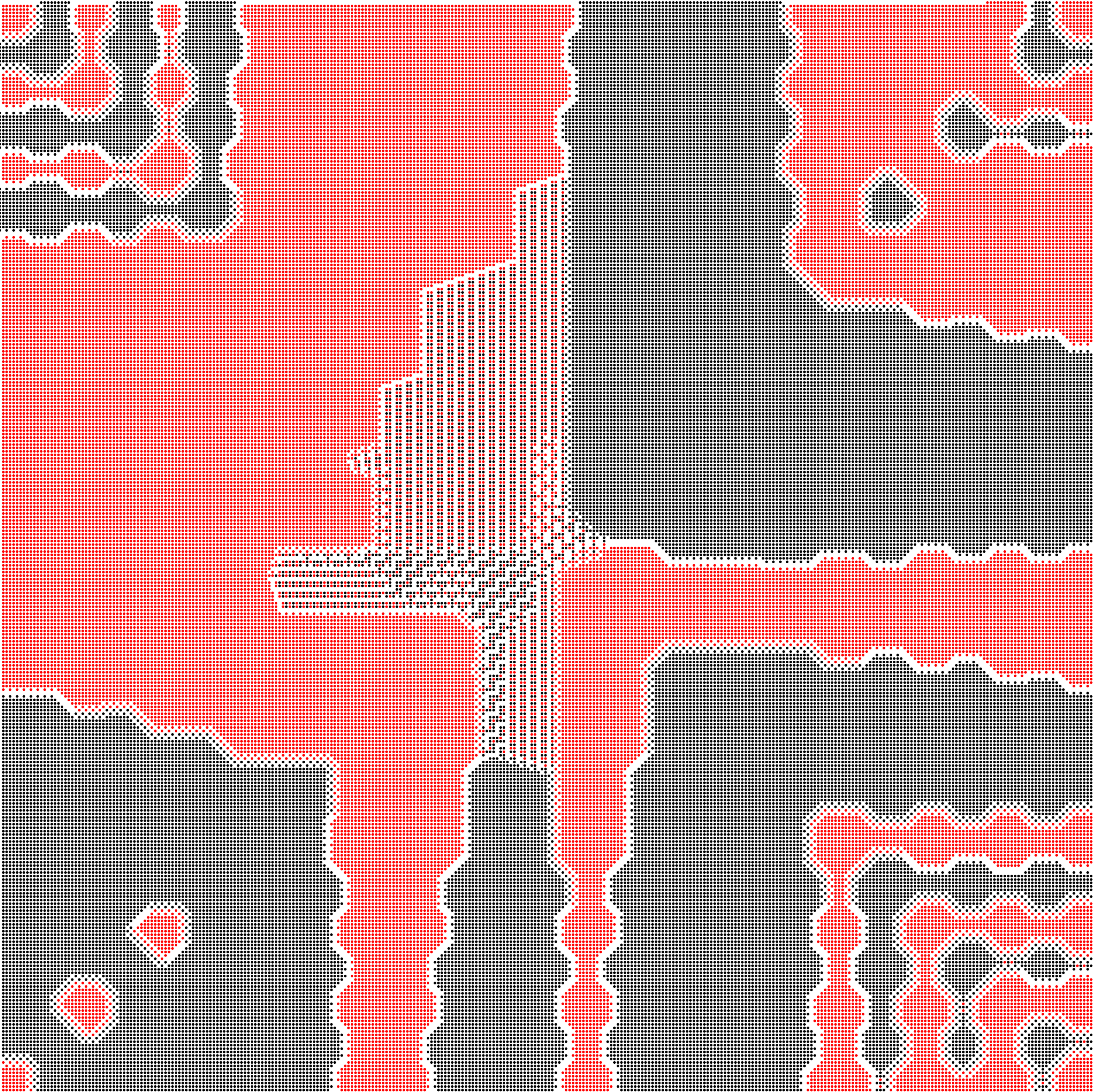
On the square spiral, alternately place Black and Red knights, always choosing smallest-numbered square not attacked (or occupied) by a knight of the opposite color.

Squares occupied by black (resp. red) knights.



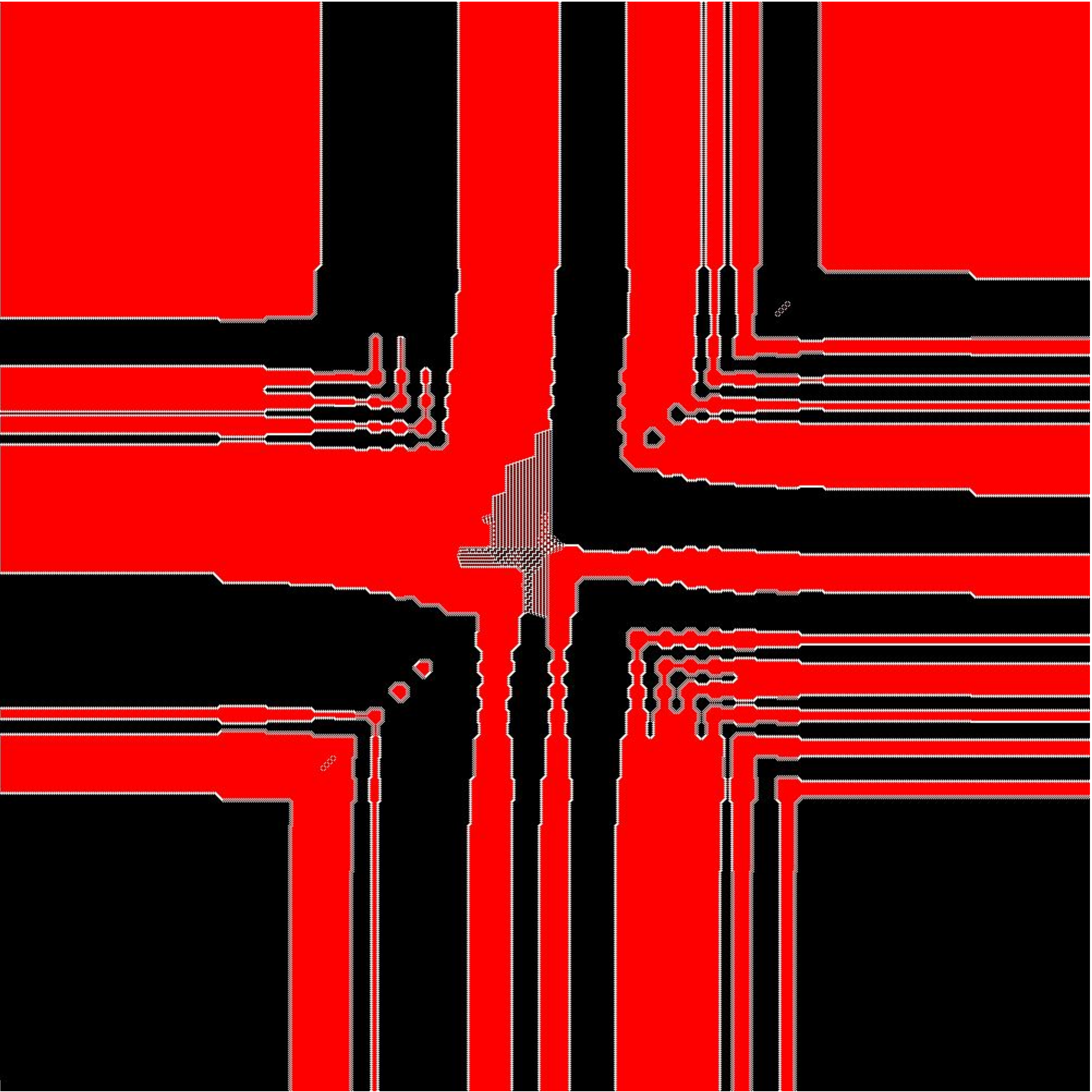
10³ steps
(Jonas Karlsson)

A392177 (B)
A392178 (R)



10^5 steps
(Jonas Karlsson)

A392177 (B)
A392178 (R)

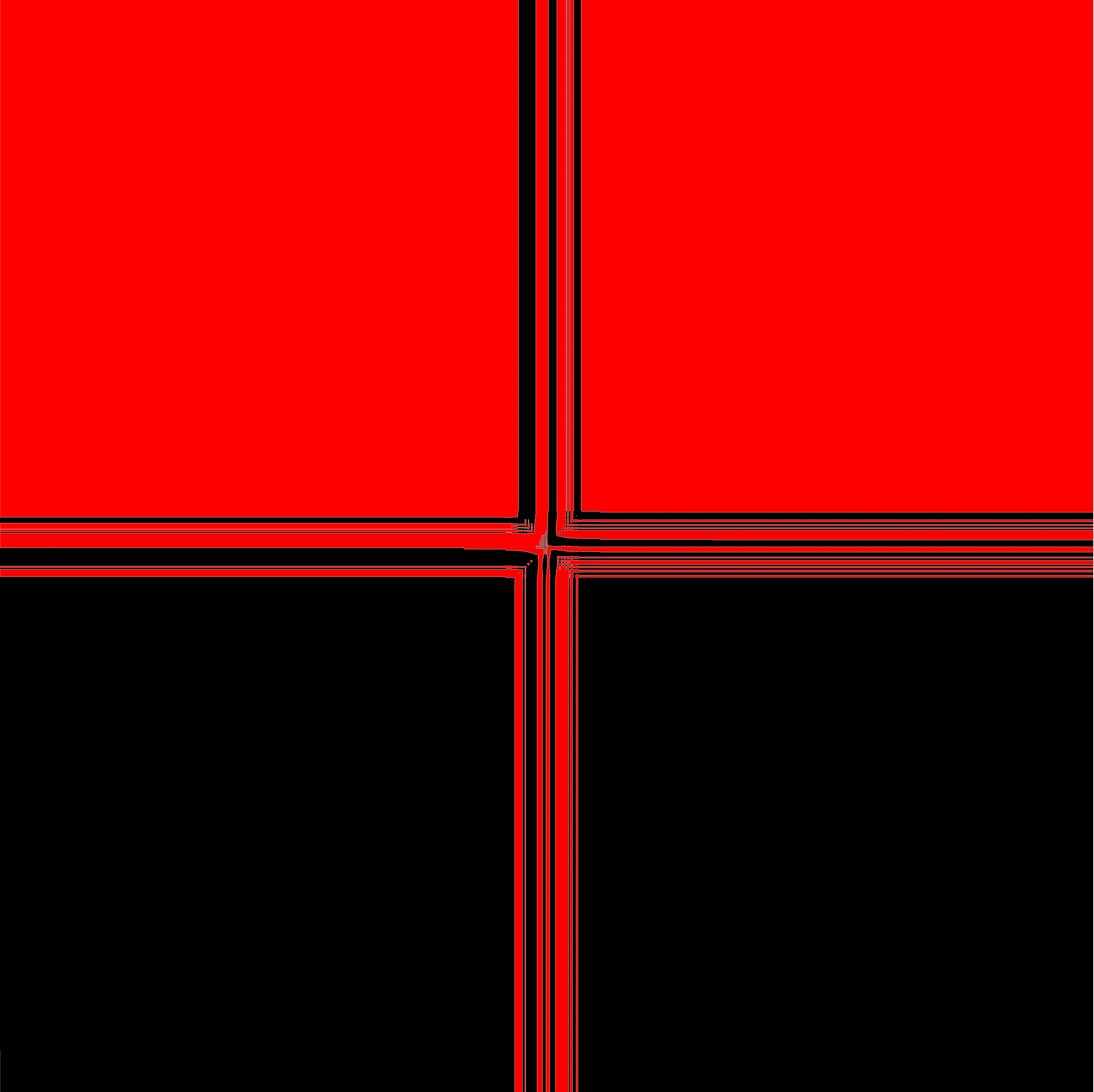


10^6 steps
(Michael Branicky)

Red and Black Knights (cont)

Most amazing
graph

$64 \cdot 10^6$ steps
(Michael Branicky)



Theorem (Branicky, Karlsson, Sloane).

**As n increases the graph converges to solid red in QI and QII, and solid black in QIII and QIV.
There are vanishingly small mixed-color strips along the four axes.**

**Useful transforms for analyzing sequences
(Maple + Unix)**

DIFF, RUNS, RECORDS

\$ cat a392177_3.txt | tr ", " " " | tr "\\ " " " | DIFF.awk | RUNS.awk | RECORDS.awk >out5d

\$ cat out5d | awk '{print \$2}' | DIFF.awk >out5e

which at line 58 settles down to

42

76

42

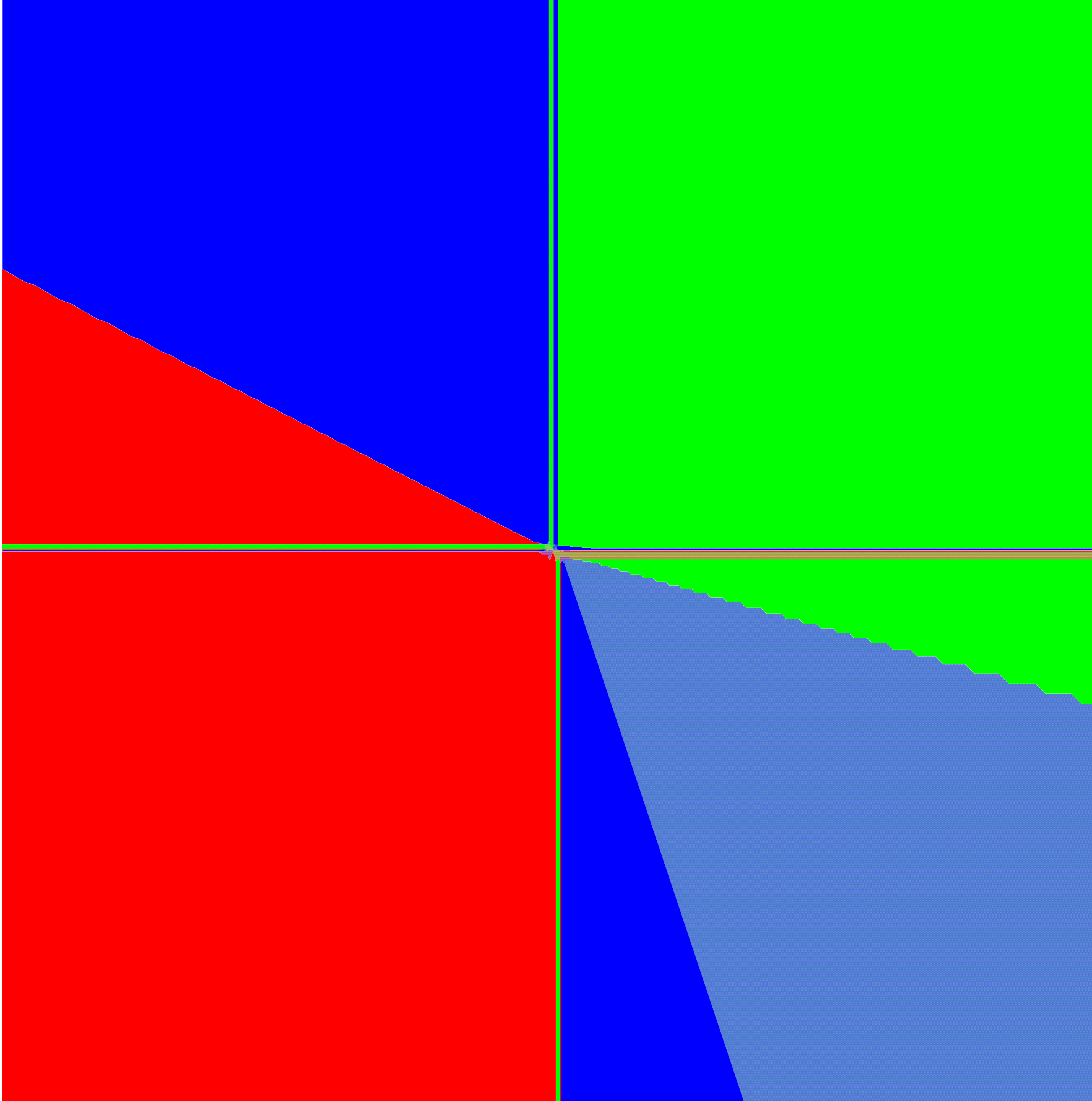
76

...

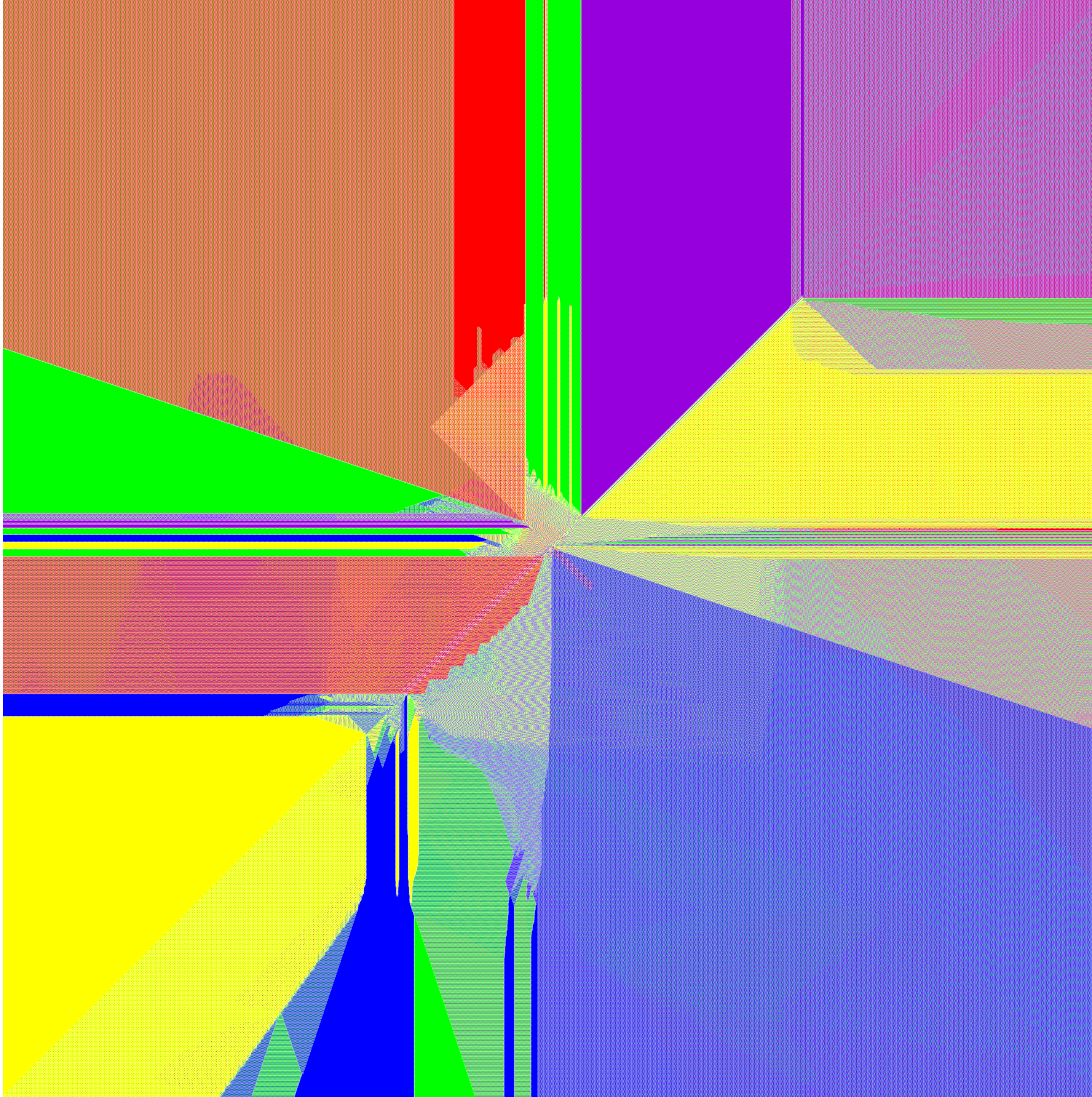
and repeats (for ever)

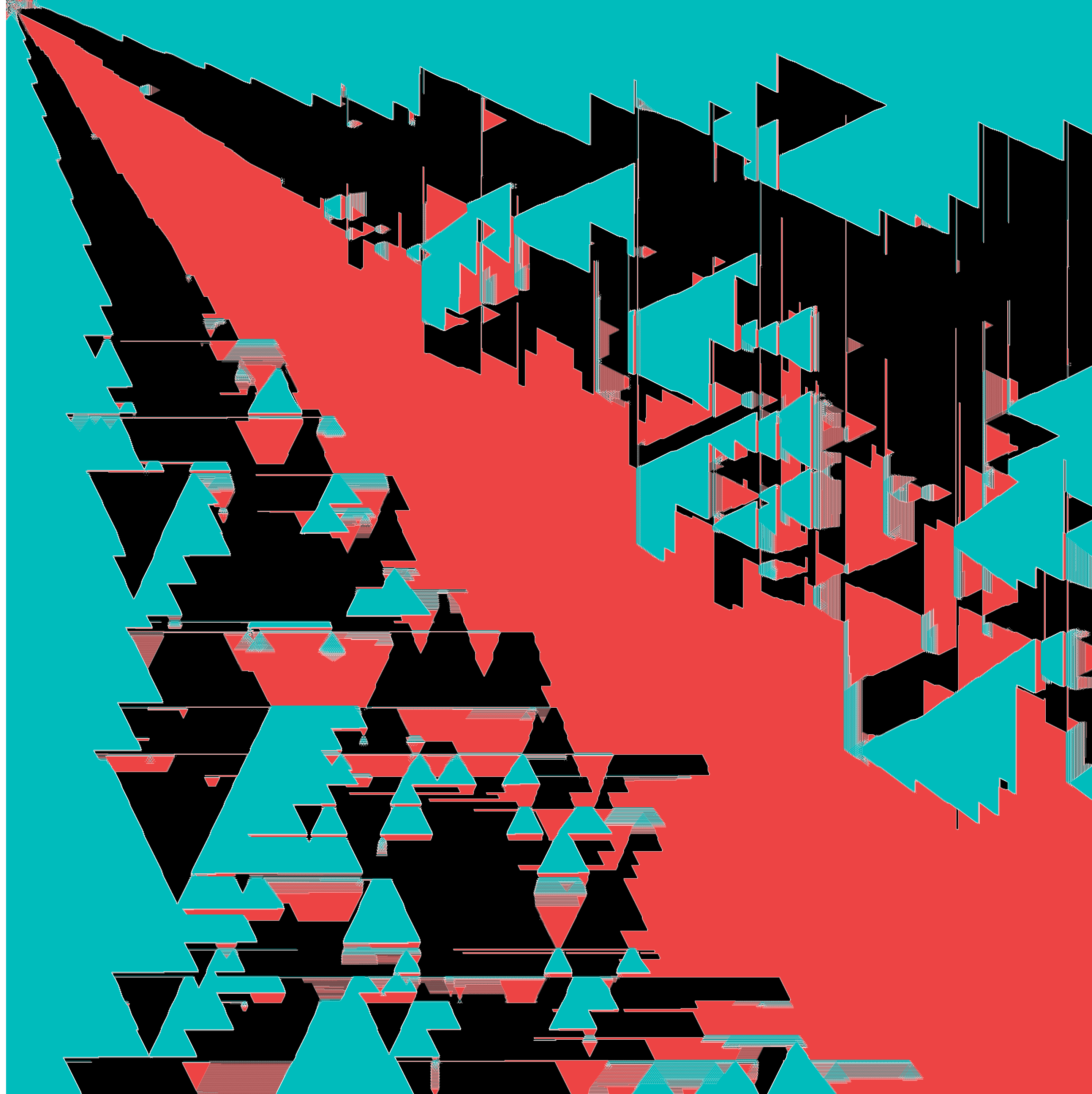
**We now know that all the irregularities end after 33591553 steps, at about shell 4112.
For the full proof of the theorem, see "Stendhal's Revenge" (in preparation).**

Michael S. Branicky
Knights of 3 colors,
 10^8 cells (A396645)



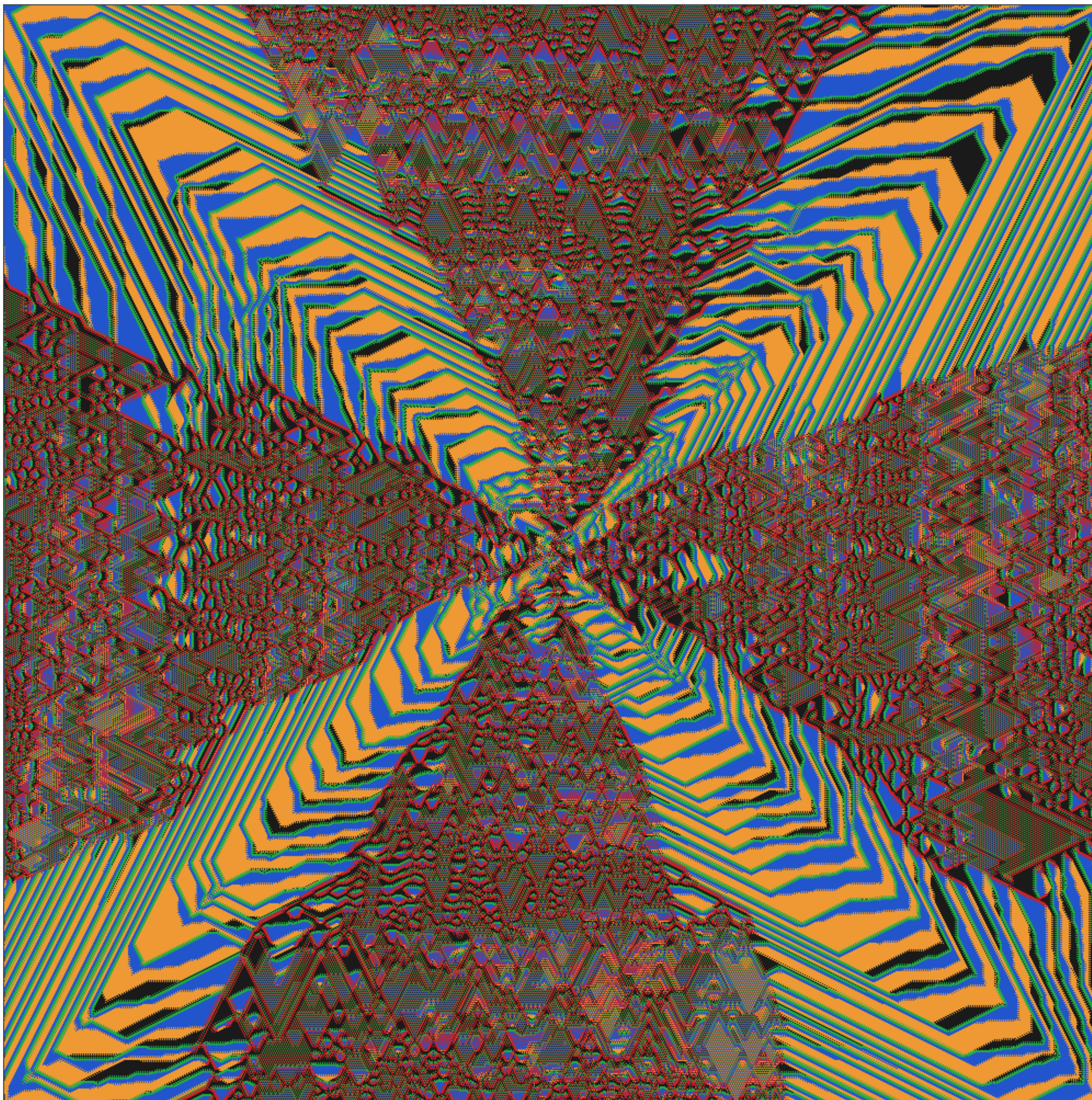
Scott R. Shannon
Knights of 5 colors,
 10^7 cells (A396122)





**Sander Huisman,
single quadrant,
knight, antelope, ferz,
 10^7 cells (A396462)**

**ferz = (1,1) leaper,
chess knight = (1, 2) leaper,
antelope = (3, 4) leaper**



Yakim Pirozhenko,
email of May 27 2026