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Combinatorics I, Dr. Z. , **Problem Set #1**, Due Nov. 14, 2002

1. Without computer, find how many integer partions of 20 are there.
2. Without computer, find how many compositions of 20 are there.
3. How many ways to arrange 8 Russian dolls?
4. Define a *looloo* to be an ordered *triple* of *moomoo*s, where a *moomoo* is a vector of odd length each of whose entries are 2 by 2 matrices with with odd row- and column- sums. Define the *magnitude* of a looloo to be the some of the entries of all its constituent members. Find $\sum_{n=0} a(n)x^n$ where $a(n)$ is the number of creatures of looloos of magnitude n
5. How many ordered trees are there with 5 vertices with 3 children, 7 vertices with 2 children, 9 vertices with 1 child and 4 leaves, and none others?
6. Use the generatingfunctionlogy methodology to prove that $\sin^2 z + \cos^2 z = 1$.
7. An up-down permutation is a permutation $\pi_1 \dots \pi_n$ such that $\pi_1 < \pi_2 > \pi_3 < \pi_4$ .... e.g. 1423 is an up-down perm. Let $a(n)$ be the number of up-down permutations. Find

$$\sum_{n=0}^{\infty} \frac{a(n)}{n!} x^n$$

8. (i) For the symmetric group of 4 elements, find all the subgroups. (ii) For each of these, write the cycle-index polynomial.
9. In how many ways can you color the faces of a (i) tetrahedral die with 2 blue and 2 red faces (ii) cubical die with 3 blue, 2 green, and one red face (iii) Octahedral die with 2 blue, 2 green, and 4 red faces
10. Use Polya theory to find how many unlabelled graphs are there with 6 vertices?
11. For the Boolean lattice B_n, let

$$X(S) := \sum_{i \notin S} [S \cup \{i\}] \quad , Y(S) := \sum_{i \in S} [S - \{i\}] \quad .$$

Find $XY - YX$. Use it to prove that X is injective for $|S| < n/2$.

12. What's is the average number of fixed points of a permutation on $\{1, \ldots, n\}$?
13. Without looking, reproduce the Kahn-Kalai disproof of the Borsuk conjecture