Solutions to Dr. Z.'s Math 421 REAL Quiz #7

1. (5 points) The **Zoe** polynomials $Z_n(x)$ are defined by

$$Z_n(x) = Z_{n-1}(x) + Z_{n-2}(x) + xZ_{n-3}(x)$$

with initial conditions $Z_0(x) = 1, Z_1(x) = x, Z_2(x) = x^2$. Find $Z_3(x)$ and $Z_4(x)$.

Sol. When $n = 3$:

$$Z_3(x) = Z_{3-1}(x) + Z_{3-2}(x) + xZ_{3-3}(x) = Z_2(x) + Z_1(x) + xZ_0(x) = x^2 + x + x(1) = x^2 + 2x$$

When $n = 4$:

$$Z_4(x) = Z_3(x) + Z_2(x) + xZ_1(x) = x^2 + 2x + x^2 + x(x) = x^2 + 2x + x^2 + x^2 = 3x^2 + 2x \quad .$$

Ans. to 1.: $Z_3(x) = x^2 + 2x$, $Z_4(x) = 3x^2 + 2x$.

2. (5 points) Find product solutions, if possible, to the partial differential equation

$$\frac{\partial^2 u}{\partial x^2} - 2\frac{\partial^2 u}{\partial x \partial y} + \frac{\partial^2 u}{\partial y^2} = 0 \quad .$$

Sol.: We try:

$$u(x, y) = X(x)Y(y) \quad .$$

Now:

$$\frac{\partial^2 u}{\partial x^2} = X''(x)Y(y) \quad ,$$

$$\frac{\partial^2 u}{\partial x \partial y} = X'(x)Y'(y) \quad ,$$

$$\frac{\partial^2 u}{\partial y^2} = X(x)Y''(y) \quad .$$

Plugging this in the pde we get:

$$X''Y - 2X'Y' + XY'' = 0 \quad .$$

Dividing by XY, we get:

$$\frac{X''}{X} - 2\frac{X'}{X}\frac{Y'}{Y} + \frac{Y''}{Y} = 0 \quad .$$

Now we are **stuck**. There is no way to **separate** the $X(x)$ (and x) -stuff from the $Y(y)$ (and y) stuff.

Ans. to 2: The pde is **inseparable**. It is **impossible** to use separation of variables.