Solutions to Dr. Z.'s Math 421 REAL Quiz #3

1. Solve the IVP

$$y'(t) - y(t) = \delta(t-2) \quad , \quad y(0) = 3 \quad .$$

Sol. of 1: Applying the **Laplace Transform** $\mathcal{L}$, we have

$$\mathcal{L}(y'(t)) - \mathcal{L}(y(t)) = L(\delta(t-2)) \quad , \quad y(0) = 3 \quad .$$

Let, as usual $\mathcal{L}(y(t)) = Y(s)$ (Y for short). We have

$$sY - y(0) - Y = e^{-2s} \quad .$$

Since $y(0) = 3$, we have

$$sY - 3 - Y = e^{-2s} \quad .$$

So

$$(s-1)Y = 3 + e^{-2s} \quad .$$

Solving for Y we have

$$Y = \frac{3 + e^{-2s}}{s-1} = \frac{3}{s-1} + \frac{e^{-2s}}{s-1} \quad .$$

Applying $\mathcal{L}^{-1}$

$$y = \mathcal{L}^{-1}(\frac{3}{s-1}) + +\mathcal{L}^{-1}(\frac{e^{-2s}}{s-1}) \quad .$$

$$3\,e^t + e^{t-2}\mathcal{U}(t-2) \quad .$$

Note that for the second piece we used the formula $\mathcal{L}^{-1}(e^{-as}F(s)) = f(t-a)\mathcal{U}(t-a)$, where $F(s) = \frac{1}{s-1}$ and $a = 2$.