Solution to Dr. Z.'s Math 421, REAL Quiz #2

1. Compute

$$\mathcal{L}^{-1}\left\{\frac{1}{s^3+9s}\right\} \quad .$$

Sol.: First **factorize!**:

$$\frac{1}{s^3+9s} = \frac{1}{(s^2+9)s} \quad .$$

Next, set up a (correct!) **template** for a partial-fraction decomosition:

$$\frac{1}{s^3+9s} = \frac{1}{(s^2+9)s} = \frac{A}{s} + \frac{Bs+C}{s^2+9} \quad .$$

Taking **common denominator**:

$$\frac{1}{(s^2+9)s} = \frac{A}{s}+\frac{Bs+C}{s^2+9} = \frac{A(s^2+9)+(Bs+C)s}{(s^2+9)s} = \frac{As^2+9A+Bs^2+Cs}{(s^2+9)s} = \frac{(A+B)s^2+Cs+9A}{(s^2+9)s} \quad .$$

Equating the numerators:

$$1 = (A+B)s^2+Cs+9A \quad .$$

Comparing coefficients of s^2, s, and 1:

$$A+B=0 \quad , \quad C=0 \quad , \quad 9A=1 \quad .$$

So $A=\frac{1}{9}, B=-\frac{1}{9}, C=0$. Going back to the template, we get:

$$\frac{1}{s^3+9s} = \frac{\frac{1}{9}}{s} - \frac{\frac{1}{9}s}{s^2+9} = \frac{1}{9}\cdot\frac{1}{s} - \frac{1}{9}\cdot\frac{s}{s^2+9} \quad .$$

Finally, taking $\mathcal{L}^{-1}$ plus a table look-up:

$$\mathcal{L}^{-1}\{\frac{1}{s^3+9s}\} = \frac{1}{9}\mathcal{L}^{-1}\{\frac{1}{s}\} - \frac{1}{9}\mathcal{L}^{-1}\{\frac{s}{s^2+9}\} = \frac{1}{9}\cdot 1 - \frac{1}{9}\cos 3t = \frac{1}{9}(1-\cos 3t) \quad .$$

Ans. to 1: $\frac{1}{9}(1-\cos 3t)$.

Comments: 1. Quite a few people chose the wrong template for the partial-fraction decomposition, for example $\frac{A}{s}+\frac{B}{s^2+9}$. Once you do that, you are doomed. Please make sure that you know the right template to use.

2. Another way of doing it is via the formula from Lecture 4 (but Lecture 4 was not part of this quiz)

$$\mathcal{L}^{-1}\{\frac{F(s)}{s}\} = \int_0^t f(\tau)\,d\tau \quad ,$$

with $F(s)=\frac{1}{s^2+9}$, but one has to be careful not to mess up the integration!