Solutions to Dr. Z.'s Math 421 Quiz #1

1. Using the **definition** find the Laplace transform $\mathcal{L}\{f(t)\}$ (alias $F(s)$) of $f(t) = e^{-t} + 3$.

Solution of 1];

$$\mathcal{L}(e^{-t}+3) = \int_{t=0}^{\infty} (e^{-t}+3)e^{-st}\,dt \;=\; \int_{t=0}^{\infty} (e^{-t(s+1)} + 3e^{-st})\,dt \;=\; \frac{e^{-t(s+1)}}{-(s+1)}\Big|_0^{\infty} + \frac{3e^{-ts}}{-s}\Big|_0^{\infty}$$

$$\frac{e^{-\infty}}{-(s+1)} - \frac{e^{0}}{-(s+1)} + \frac{e^{-\infty}}{-s} - \frac{3e^{0}}{-s}$$

$$\frac{1}{s+1} + \frac{3}{s} \quad .$$

Ans. to 1: $\frac{1}{s+1} + \frac{3}{s}$ **.**

2. Using Tables, find $\mathcal{L}\{f(t)\}$, if $f(t) = (t+1)(t-1) + 3t$.

Sol. to 2: First let's simplify $f(t)$.

$f(t) = (t+1)(t-1) + 3t = t^2 - 1 + 3t = t^2 + 3t - 1$

Hence

$$\mathcal{L}(f(t)) = \mathcal{L}(t^2+3t-1) = \mathcal{L}(t^2) + 3\mathcal{L}(t) - \mathcal{L}(1) = \frac{2!}{s^3} + 3\frac{1!}{s^2} - \frac{1}{s} = \frac{2}{s^3} + \frac{3}{s^2} - \frac{1}{s} \quad .$$

Ans. to 2: $\frac{2}{s^3} + \frac{3}{s^2} - \frac{1}{s}$ **.**