

**Solutions to Dr. Z.'s Math 421(2), (Fall 2024, RU) REAL Quiz #10**

**1.** (10 points) Find the general expression, in polar coordinates, for the steady-state temperature  $u(r, \theta)$  in the **infinite** plane with a circular hole of radius 2 cut-out, and where the temperature at the bounding ring is  $u(2, \theta) = 2 \sin 3\theta, 0 < \theta < 2\pi$ .

**Sol.** The boundary function  $f(\theta) = 2 \sin 3\theta$  is its own Fourier Series, so all we need is multiply each  $\sin n\theta$  and/or  $\cos n\theta$  by  $(r/c)^{-n}$  (since this is an *infinite* plate, we need *negative powers*).

Here there is only one pure-sine term, so the answer is simply

$$u(r, \theta) = 2(r/2)^{-3} \sin 3\theta = 16r^{-3} \sin 3\theta \quad .$$

**Comment:** Many people didn't get it. What a shame! It is such an easy problem. Please review the problem and make sure you understand how to do it.