Solutions to the Attendance Quiz/Diagnostic Review Test for Lecture -1

1. Compute $993 \cdot 1007$.

Sol. to 1: **ShortCut Way**: $993 \cdot 1007 = (1000-7)\cdot(1000+7) = 1000^2 - 7^2 = 1000000 - 49 = 999951$.

Ans. to 1: 999951.

Note: most people did it correctly the "usual" way, a few did it nicer (but not as nice as my way!) by doing

$$993 \cdot (1000+7) = 993 \cdot 1000 + 993 \cdot 7 = 993000 + 6951 = 999951 \quad .$$

2. Simplify $2x\sin^2 5x + 2x\cos^2 5x - x$.

Sol. to 2

$$2x\sin^2 5x + 2x\cos^2 5x - x = 2x(\sin^2 5x + \cos^2 5x) - x = 2x \cdot 1 - x = x \quad .$$

Here we used the famous trig. identity $\cos^2 w + \sin^2 w = 1$ (where $w = 5x$).

Ans. to 2: x.

Note: About half got it fully. Quite a few people were on the right track, but carelessly "simplified" $\sin^2 5x + \cos^2 5x$ as 5.

Another Note: A few people got it correctly but left the answer as $2x - x$. It is very impolite not to simplify $2x - x$ to x.

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3. What is the largest value that the function $f(x) = x - x^2$ can take in the interval $0 \leq x \leq 1$. At what value of x does that happen?

Sol. to 3: $f'(x) = 1 - 2x$. Solving $f'(x) = 0$ gives $1 - 2x = 0$ so $x = \frac{1}{2}$. At the endpoints $f(0) = 0, f(1) = 0$, and at the critical point, $x = \frac{1}{2}$, $f(\frac{1}{2}) = \frac{1}{2} - \frac{1}{2}^2 = \frac{1}{4}$. The largest value is $\frac{1}{4}$ so

Ans. to 3: The **largest value** of $f(x) = x - x^2$ is $\frac{1}{4}$ and it takes place at $x = \frac{1}{2}$.

4. What is the largest value that the function $f(x) = \frac{x}{e^x}$ can take when $x \geq 0$. At what value of x does that happen?

Sol. to 4: First rewrite $f(x) = xe^{-x}$ (otherwise you would have to use the quotient rule, and that is a pain).

By the product rule

$$f'(x) = x'e^{-x} + x(e^{-x})' = e^{-x} + x(-e^{-x}) = (1-x)e^{-x} \quad .$$

Setting $f'(x) = 0$ gives the equation $(1-x)e^{-x} = 0$. Since e^{-x} is never zero, we can divide by it, getting $x - 1 = 0$, so the only critical point is $x = 1$. At the endpoint $x = 0$, $f(0) = 0$ and at the other "endpoint", ∞, it is also zero. (More precisely $\lim_{x\to\infty} \frac{x}{e^x} = 0$). So the largest **value** is $f(1) = e^{-1} = \frac{1}{e}$, and it takes place at $x = 1$.

Ans. to 4 The largest **value** of $f(x) = \frac{x}{e^x}$ is $\frac{1}{e}$ and it takes place at $x = 1$.

Note: About %50 got it right. Quite a few people messed up the differentiation.

5. Evaluate $\int_0^1 x\, e^x\, dx$.

Sol. to 5: We use the **integration by parts** formula

$$\int uv'\, dx = uv - \int u'v\, dx \quad ,$$

with $u = x$ and $v = e^x$, getting

$$\int xe^x\, dx = xe^x - \int x'e^x\, dx = xe^x - \int e^x\, dx = xe^x - e^x = (x-1)e^x \quad .$$

This was the **indefinite integral**. Now we stick-in **limits of integration**:

$$\int_0^1 xe^x\, dx = (x-1)e^x|_0^1 = (1-1)e^1 - (0-1)e^0 = 0 - (-1) = 1 \quad .$$

Ans. to 5.: 1 .

Note: Only about %30 got it completely right. Some people are rusty with integration-by-parts. Other people got the integration part right, but messed up the plugging-in part, and the subsequent arithmetics, getting, for example, -1.

6. If $f(x, y, z) = xyz$, find its gradient, **grad** f.

Sol. to 6:

$$\mathbf{grad}\, f = < \frac{\partial f}{\partial x}, \frac{\partial f}{\partial y}, \frac{\partial f}{\partial z} > = < yz, xz, xy > \quad .$$

Ans. to 6: $\mathbf{grad}\, f = < yz, xz, xy >$

Note: About %60 of the students got it right. A very common error was $yz + xz + xy$, this is **wrong**!. The **output** is a vector (field), not a single function!

7. Find a function $y(x)$ such that

$$y''(x) + y(x) = 0 \quad , \quad y(0) = 1 \quad , \quad y'(0) = 1 \quad .$$

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Sol. to 7: The **auxiliary equation** of this linear, constant-coefficients, homogeneous ode is

$$r^2 + 1 = 0$$

Solving gives the complex roots $r = 0 \pm i$, so the **general solution** is

$$y(x) = c_1 \cos x + c_2 \sin x \quad .$$

For future reference

$$y'(x) = -c_1 \sin x + c_2 \cos x \quad .$$

Plugging-in $x = 0$ gives

$$y(0) = c_1 \cos 0 + c_2 \sin 0 = c_1 \quad ,$$

$$y'(0) = -c_1 \sin 0 + c_2 \cos 0 = c_2 \quad .$$

Using the initial data, we get

$$1 = c_1 \quad , \quad 1 = c_2 \quad ,$$

so $c_1 = 1, c_2 = 1$. Going back to the general solution, we get

$$y(x) = 1 \cdot \cos x + 1 \cdot \sin x = \cos x + \sin x \quad .$$

Ans. to 7: $y(x) = \cos x + \sin x$.

Note: About %60 of the students got it right. Some people are rusty. We will soon learn another method for solving such IVPs.