Solutions to the Attendance Quiz for Dr. Z.'s Math 421 (RU, Fall 2014) Lecture 9

1. Find the half-range cosine expansion of the following function defined on $(0, \pi)$

$$f(x) = \begin{cases} 2, & \text{if } 0 \leq x < \pi/2; \\ -1, & \text{if } \pi/2 \leq x < \pi \ . \end{cases}$$

Sol. The formula to use is

$$\frac{a_0}{2} + \sum_{n=1}^{\infty} a_n \cos nx \quad ,$$

where

$$a_0 = \frac{2}{\pi} \int_0^{\pi} f(x)\, dx$$

$$a_n = \frac{2}{\pi} \int_0^{\pi} f(x) \cos nx\, dx$$

For $f(x)$ of the problem the integration has to be broken-up into two pieces, from 0 to $\pi/2$, and from $\pi/2$ to π.

$$a_0 = \frac{2}{\pi} \int_0^{\pi} f(x)\, dx = \frac{2}{\pi} \int_0^{\pi/2} f(x)\, dx + \frac{2}{\pi} \int_{\pi/2}^{\pi} f(x)\, dx = \frac{2}{\pi} \int_0^{\pi/2} (2)\, dx + \frac{2}{\pi} \int_{\pi/2}^{\pi} (-1)\, dx$$

$$= \frac{2}{\pi} 2(\pi/2) + \frac{2}{\pi}(-1)(\pi/2) = 2 - 1 = 1 \quad .$$

$$a_n = \frac{2}{\pi} \int_0^{\pi} f(x) \cos nx\, dx = \frac{2}{\pi} \int_0^{\pi/2} f(x) \cos nx\, dx + \frac{2}{\pi} \int_{\pi/2}^{\pi} f(x) \cos nx\, dx = \frac{2}{\pi} \int_0^{\pi/2} (2) \cos nx\, dx + \frac{2}{\pi} \int_{\pi/2}^{\pi} (-1) \cos nx\, dx$$

$$= \frac{4}{\pi} \left(\frac{\sin nx}{n} \Big|_0^{\pi/2} \right) - \frac{2}{\pi} \left(\frac{\sin nx}{n} \Big|_{\pi/2}^{\pi} \right) = \frac{4}{\pi} \left(\frac{\sin(n\pi/2)}{n} - 0 \right) - \frac{2}{\pi} \left(\frac{\sin n\pi}{n} - \frac{\sin n\pi/2}{n} \right)$$

$$= \frac{4}{\pi} \frac{\sin(n\pi/2)}{n} - \frac{2}{\pi} \left(0 - \frac{\sin n\pi/2}{n} \right)$$

$$= \frac{6}{\pi} \frac{\sin(n\pi/2)}{n}$$

So $a_n = \frac{6}{\pi} \frac{\sin(n\pi/2)}{n}$.

Putting it back above we get that the half-range cosine expansion of the given function, $f(x)$ is

$$\frac{1}{2} + \frac{6}{\pi} \sum_{n=1}^{\infty} \frac{\sin(n\pi/2)}{n} \cos nx \quad .$$

This is a correct answer. Since $\sin n\pi/2 = 0$ when n is even and $\sin(2k+1)/2\pi = (-1)^k$, we can make it look nicer as follows

$$\frac{1}{2} + \frac{6}{\pi} \sum_{k=1}^{\infty} \frac{(-1)^k}{(2k+1)} \cos(2k+1)x \quad .$$

Comment: About %25 of the people finished the problem, but many were on the right track.