Solutions to the Attendance Quiz for Lecture 7

1. Show that the given functions are orthogonal on the given interval.

$$f_1(x) = \sin x \quad , \quad f_2(x) = \cos x \quad , \quad [0, \pi] \quad .$$

Sol.: We have to show that

$$(f_1(x), f_2(x)) = \int_0^{\pi} \sin x \cos x \, dx = 0 \quad .$$

But by trig. identity $\sin x \cos x = \frac{1}{2} \sin 2x$, so

$$\int_0^{\pi} \sin x \cos x \, dx = \frac{1}{2} \int_0^{\pi} \sin 2x \, dx = -\frac{\cos 2x}{4} \Big|_0^{\pi} = -\frac{\cos 0 - \cos 2\pi}{4} = -\frac{1-1}{4} = \frac{0}{4} = 0 \quad .$$

2. Verify by direct integration that the functions are orthogonal with respect to the indicated weight function on the given interval

$$f(x) = x - 3 \quad , \quad g(x) = x^2 \quad , \quad w(x) = e^{-x} \quad , [0, \infty) \quad .$$

Sol. to 2:

$$(f(x), g(x))_w(x) = \int_0^{\infty} (x-3) x^2 e^{-x} \, dx = \int_0^{\infty} (x^3 - 3x^2) e^{-x} \, dx = \int_0^{\infty} x^3 e^{-x} \, dx - 3 \int_0^{\infty} x^2 e^{-x} \, dx - \quad .$$

By using the famous formula

$$\int_0^{\infty} x^n e^{-x} \, dx = n! \quad ,$$

we get that the integral equals

$$3! - 3(2!) = 6 - 3(2) = 0 \quad .$$