Solutions to the Attendance Quiz for Lecture 4

1. Find $\mathcal{L}\{t \sin 2t\}$

Sol.: We use the formula

$$\mathcal{L}\{t^n f(t)\} = (-1)^n \frac{d^n}{ds^n} F(s) \quad .$$

Here $n = 1$ and $f(t) = \sin 2t$. From the tables (or from our heads), $F(s) = \frac{2}{s^2+4} = 2(s^2+4)^{-1}$. Taking the first derivative, we have, by the **chain rule**,

$$F'(s) = (2(s^2+4)^{-1})' = 2(-1)(s^2+4)^{-2}(2s) = -4s(s^2+4)^{-2} = \frac{-4s}{(s^2+4)^2} \quad .$$

So

$$\mathcal{L}\{t \sin 2t\} = (-1)^1 F'(s) = -\frac{-4s}{(s^2+4)^2} = \frac{4s}{(s^2+4)^2} \quad .$$

Ans. to 1.: $\frac{4s}{(s^2+4)^2}$.

Comment: About %60 of the students got it completely right, another %20 were on the right track, and the remaining %20 messed up completely.

2. Evaluate

$$\mathcal{L}\{\int_0^t e^{4\tau} \sin(5(t-\tau))\, d\tau\} \quad .$$

Sol. of 2.; The integral is nothing but the **convolution**

$$e^{4t} * \sin 5t \quad .$$

Using the formula $\mathcal{L}\{f(t) * g(t)\} = F(s)G(s)$, we have

$$\mathcal{L}\{\int_0^t e^{4\tau} \sin(5(t-\tau))\, d\tau\} = \mathcal{L}\{e^{4t} * \sin 5t\} = \mathcal{L}\{e^{4t}\}\mathcal{L}\{\sin 5t\} \quad .$$

From the tables, $\mathcal{L}\{e^{4t}\} = \frac{1}{s-4}$, $\mathcal{L}\{\sin 5t\} = \frac{5}{s^2+25}$. So we get that the answer is

$$\frac{1}{s-4}\frac{5}{s^2+25} = \frac{5}{(s-4)(s^2+25)} \quad .$$

Ans. to 2.: $\frac{5}{(s-4)(s^2+25)}$.

Comment: About %70 of the students got it completely right, another %15 were on the right track, and the remaining %15 messed up completely, or were clueless.