Solutions to the Attendance Quiz #2

1. Find $\mathcal{L}^{-1}\{\frac{3s+1}{s^2+25}\}$

Sol.: We first break-up the **numerator**:

$$\frac{3s+1}{s^2+25} = \frac{3s}{s^2+25} + \frac{1}{s^2+25} = 3\frac{s}{s^2+25} + \frac{1}{s^2+25}$$

We next consult the **tables**:

$$\mathcal{L}^{-1}\{\frac{s}{s^2+k^2}\} = \cos kt \quad , \quad \mathcal{L}^{-1}\{\frac{1}{s^2+k^2}\} = \frac{\sin kt}{k} \quad .$$

with $k = 5$ and get that

$$\mathcal{L}^{-1}\{\frac{3s+1}{s^2+25}\} = 3\mathcal{L}^{-1}\{\frac{s}{s^2+25}\} + \mathcal{L}^{-1}\{\frac{1}{s^2+25}\} = 3\cos 5t + \frac{1}{5}\sin 5t \quad .$$

Ans. to 1.: $3\cos 5t + \frac{1}{5}\sin 5t$.

Comments: About %70 of the students got the right answer. Some people tried to factorize the bottom $s^2 + 25$, but of course, this is not possible, since the roots of $s^2 + 25 = 0$ are complex numbers.

2. Evaluate

$$\mathcal{L}^{-1}\{\frac{2s-1}{s^2-s-2}\}$$

Sol.: **Now** it is possible to factorize the bottm: $s^2 - s - 2 = (s-2)(s+1)$, so

$$\frac{2s-1}{s^2-s-2} = \frac{2s-1}{(s-2)(s+1)} \quad .$$

We use the **template**:

$$\frac{2s-1}{(s-2)(s+1)} = \frac{A}{s-2} + \frac{B}{s+1} \quad .$$

Taking common-denominator of the right side:

$$\frac{2s-1}{(s-2)(s+1)} = \frac{A}{s-2} + \frac{B}{s+1} = \frac{A(s+1)+B(s-2)}{(s-2)(s+1)} \quad .$$

The denominators automatically match, but to make the numerators agree we must have

$$2s - 1 = A(s+1) + B(s-2) \quad .$$

We now plug-in **convenient values**: $s = -1$ gives

$$2(-1) - 1 = A(-1+1) + B(-1-2) = -3B \quad ,$$

so $-3 = -3B$ and we have $B = 1$. $s = 2$ gives

$$2(2) - 1 = A(2+1) + B(2-2) = 3A \quad ,$$

so $3 = 3A$ and we have $A = 1$. Having found $A = 1$ and $B = 1$ we plug them into the template and get

$$\frac{2s-1}{(s-2)(s+1)} = \frac{1}{s-2} + \frac{1}{s+1} \quad .$$

Now it is time to apply $\mathcal{L}^{-1}$:

$$\mathcal{L}^{-1}\{\frac{2s-1}{(s-2)(s+1)}\} = \mathcal{L}^{-1}\{\frac{1}{s-2}\} + \mathcal{L}^{-1}\{\frac{1}{s+1}\} = e^{2t} + e^{-t} \quad ,$$

where we used the table-formula

$$\mathcal{L}^{-1}\{\frac{1}{s-a}\} = e^{at} \quad ,$$

with $a = 2$ and $a = -1$ respectively.

Ans. to 2.: $e^{2t} + e^{-t}$.

Comments: About %75 of the students got the right answer. Another %10 were on the right track but made careless mistakes, for example a few people got the incorrect answer $e^{2t} + e^{t}$. Remember $\mathcal{L}^{-1}\{\frac{1}{s+1}\} = e^{-t}$ since $a = -1$ in the formula.