Solutions to the Attendance Quiz Lecture 18

1. Find the general expression, in polar coordinates, for the steady-state temperature $u(r, \theta)$ in a circular plate of radius 2, if the temperature on the circumference $r = 2$ is given by $u(2, \theta) = \sin\theta$.

Sol. Remember that when $u(c, \theta)$ is a **pure** sine-wave of the form $C \sin n\theta$ for some **integer** n, then the answer is simply $C(r/c)^n \sin n\theta$.

In this problem $c = 2$ $(C = 1)$ and $n = 1$ so $u(r, \theta) = (r/2)^1 sin\theta = (r/2)\sin\theta$

Ans. to 1: $u(r, \theta) = \frac{r \sin\theta}{2}$.

Comments: **1.** Almost everyone got it right. A few people went further and "simplified" the answer to $(2/2)\sin\theta = \sin\theta$. This is **wrong**! $c = 2$ **but** r is *not* 2, r is a variable!

2. If $u(c, \theta)$ is a finite combination of pure sine and/or pure cosine waves, you do the same to each term individually.

For example, if $u(2, \theta) = 3\sin 2\theta - 5\sin 4\theta + 7\cos 2\theta$, the answer would be

$$u(r, \theta) = 3(r/2)^2 \sin 2\theta - 5(r/2)^4 \sin 4\theta + 7(r/2)^2 \cos 2\theta \quad .$$

3. If $u(r, \theta)$ is an **infinite** combination of pure sine and pure cosine waves, then you still do the **same thing**. Alas, in this case there is a preliminary step of expressing the given function as a combination of pure sine and pure cosines waves (also known as finding the full Fourier Series on $(-\pi, \pi)$), and then doing the above process of sticking $(r/c)^n$ in front of $\sin n\theta$ and $\cos n\theta$ (if applicable).