Solutions to the Attendance Quiz for Lecture 15

1. Using the **ready-made formula** (don't do it from scratch) solve the boundary value problem

$$4\frac{\partial^2 u}{\partial x^2} = \frac{\partial u}{\partial t} \quad , \quad 0 < x < \pi \quad , \quad t > 0 \quad ,$$

$$u(0,t) = 0 \quad , \quad u(\pi,t) = 0 \quad , t > 0$$

$$u(x,0) = 3x(\pi - x) \quad , \quad 0 < x < \pi \quad ,$$

Hint (from Maple):

$$\int_0^\pi x(\pi - x)\sin nx\, dx = \frac{2(1-(-1)^n)}{n^3} \quad .$$

Sol. The general formula, from the cheatsheet is (since $k = 4$ and $L = \pi$) is

$$u(x,t) = \sum_{n=1}^{\infty} A_n e^{-4n^2 t} \sin nx \quad ,$$

where

$$A_n = \frac{2}{\pi}\int_0^\pi f(x)\sin nx\, dx \quad .$$

We have to compute A_n.

$$A_n = \frac{6}{\pi}\int_0^\pi x(\pi - x)\sin nx\, dx = \frac{6}{\pi}\cdot\frac{2(1-(-1)^n)}{n^3} = \frac{12(1-(-1)^n)}{\pi n^3} \quad .$$

Putting it above:

$$u(x,t) = \sum_{n=1}^{\infty} \frac{12(1-(-1)^n))}{\pi n^3} e^{-4n^2 t}\sin nx \quad .$$

This is a **correct** answer, but we can simplify it as follows. When n is even $(1-(-1)^n))$ is always zero, when n is odd, it is always 2, so writing $n = 2k+1$ $(k = 0, 1, 2, \ldots)$:

$$u(x,t) = \sum_{k=0}^{\infty} \frac{24}{\pi(2k+1)^3} e^{-4(2k+1)^2 t}\sin(2k+1)x \quad .$$

and this is a **better answer**. Finally, we have the freedom to rename k n, so yet-another way of writing the same answer is:

$$u(x,t) = \sum_{n=0}^{\infty} \frac{24}{\pi(2n+1)^3} e^{-4(2n+1)^2 t}\sin(2n+1)x \quad .$$

Comment: About %30 got it completely. Many people forgot to take $k = 4$ and gave the answer as: $u(x,t) = \sum_{n=0}^{\infty} \frac{24}{\pi(2n+1)^3} e^{-k(2n+1)^2 t}\sin(2n+1)x$

(or the unsimplified version). Remember the general formula has k, but each specific problem has its own (numerical) k, and **you** have to figure it out.