Solutions to the Attendance Quiz for Lecture 14

1. A string of length 10 meters coincides with the interval $[0, 10]$ on the x-axis. Set up the boundary-value problem for the displacement $u(x, t)$.

a. The ends are secured to the x-axis. The string is released from rest from the initial displacement $x^2(10 - x)^7$.

b. The ends are secured to the x-axis. The string is along the x-axis at the very beginning, but has initial velocity $sin(\pi x/10)$.

c. The right end is secured to the x-axis, but the left end moves in a **transversal** manner according to $\sin(4\pi t)$. Initially the string is undisplaced and is at rest.

Sol. The *pde* is the same in all cases:

$$\alpha^2 \frac{\partial^2 u}{\partial x^2} = \frac{\partial^2 u}{\partial t^2} \quad , \quad 0 < x < 10 \quad , \quad t > 0 \quad .$$

a.: $u(0, t) = 0, u(10, t) = 0$ (since both ends are secured to the x-axis). $u(x, 0) = x^2(10 - x)^7$ (that being the initial (transversal) displacement), $u_t(x, 0) = 0$ (since the string starts at **rest**).

b.: $u(0, t) = 0, u(10, t) = 0$ (since both ends are secured to the x-axis). $u(x, 0) = 0$ (since the string is along the x-axis, the initial (transversal) displacement is **zero**), $u_t(x, 0) = sin(\pi x/10)$ (since the initial velocity is $sin(\pi x/10)$).

c.: $u(10, t) = 0$ (since the right end is secured to the x-axis), $u(0, t) = \sin(4\pi t)$ (since the left end is moving with that velocity (in a transversal way)), $u(x, 0) = 0$ (since the string is initially undisplaced) $u_t(x, 0) = 0$ (since the string starts at **rest**).