Solutions to the Attendance Quiz 0

1. Approximate, with mesh-size $h = 1$, the solution of the boundary-value problem

$$u_{xx} + u_{yy} = 0 \quad , \quad 0 < x < 2 \quad , \quad 0 < y < 2 \quad ;$$

subject to the boundary conditions

$$u(0, y) = 2 \quad , \quad 0 < y < 2 \quad ; \quad u(2, y) = 3 \quad , \quad 0 < y < 2 \quad ;$$

$$u(x, 0) = 1 \quad , \quad 0 < x < 2 \quad ; \quad u(x, 2) = x \quad , \quad 0 < x < 2 \quad .$$

Sol.: The **boundary points** are

$(1, 0)$ (that lies on the **bottom** side), and $u_{10} = u(1, 0)$

$(0, 1)$ (that lies on the **left** side), and $u_{01} = u(0, 1)$

$(1, 2)$ (that lies on the **top** side), and $u_{12} = u(1, 2)$

$(2, 1)$ (that lies on the **right** side), and $u_{21} = u(2, 1)$

Using the **boundary conditions** we have

$u_{10} = u(1, 0) = 1$

$u_{01} = u(0, 1) = 2$

$u_{12} = u(1, 2) = 1$

$u_{21} = u(2, 1) = 3$

There is only one **interior** point $(1, 1)$ and its value there is called $u_{1,1}$. It gives rise to the equation

$$u_{11} = \frac{u_{10} + u_{01} + u_{12} + u_{21}}{4}$$

in the one unknown, u_{11}.

Using the above values for the boundary points we get:

$$u_{11} = \frac{1 + 2 + 1 + 3}{4} = \frac{7}{4} \quad .$$

Ans. $u(1, 1) \approx \frac{7}{4}$.

Comments:

1. About %80 of the students got it perfectly. Those who didn't, **please** go over the handout and this solution and understand it really well.

2. Quite a few people did it almost perfectly, *but* they wrote $u(1,1) = \frac{7}{4}$, instead of the **correct**: $u(1,1) \approx \frac{7}{4}$. This numerical method (called the method of Finite Differences) only gives you *approximations*, and since here h is so big, this happens to be a very bad approximation, and the point of the problem is to teach you the method, but in real life it is done by computers, with a much smaller h (e.g $h = 0.1$ or even $h = 0.01$ and then the approximations are very good.

3. Some people took $u(1,2) = x$ and got an expression for $u(1,1)$ that involves x. This is **GARBAGE**! $u(1,1)$ equals (or rather approximately equals) a **NUMBER** not an expression in x. They got mixed up because the boundary values on all the other sides were constants and only on $y = 2$ it was an expression in x: $u(x,2) = x$. But the right thing to do is to realize that at the point $(1,2)$, the x-coordinate equals 1, so you plug-in $x = 1$ into the expression describing $u(x,2)$, namely x, and get $u(1,2) = 1$.