Solutions to MATH 421 (2), Dr. Z.'s , Exam 2, Tue. Nov. 26. 2024 10:20-11:40am, SEC 117

1. (15 pts.) Solve (from scratch!) the boundary value problem

$$\frac{\partial^2 u}{\partial x^2} - 3u = \frac{\partial u}{\partial t} \quad , \quad 0 < x < \pi \quad , \quad t > 0 \quad ,$$

subject to

$$u_x(0,t) = 0 \quad , \quad u_x(\pi,t) = 0 \quad , t > 0$$

$$u(x,0) = \cos 9x \quad , \quad 0 < x < \pi \quad .$$

Ans.: $u(x,t) = \cos(9x)e^{-84t}$

Sol.: We first look for **product solutions** of the **template**

$$u(x,t) = X(x)T(t) \quad .$$

Putting into the pde, we get

$$X''T - 3XT = XT' \quad .$$

Dividing by XT:

$$\frac{X''T - 3XT}{XT} = \frac{XT'}{XT} \quad .$$

Algebra:

$$\frac{X''}{X} - 3 = \frac{T'}{T} \quad .$$

Moving the 3 to the right (for convenience)

$$\frac{X''(x)}{X(x)} = 3 + \frac{T'(t)}{T(t)} \quad .$$

As usual, the left side does not depend on t, the right side does not depend on x, and they are **equal**, so **both** of them do not depend on x or t. In other words, they are both equal to a **constant**.
Unfortunately, that constant could be either positive, zero, or negative, so officially, we have to consider these three cases. But if the constant is positive, let's call it λ^2, then the general solution of $X''/X = \lambda^2$ is $c_1 \cosh(\lambda x) + c_2 \sinh(\lambda x)$ and this can never make

the boundary condition $u_x(0,t)=0, u_x(\pi,t)=0$ be satisfied. If the constant is zero, then $X(x)=c_1$ is a possibility. If the constant is negative then we have to solve the two odes

$$X''/X = -\lambda^2$$

$$3 + \frac{T'(t)}{T(t)} = -\lambda^2$$

Rewriting: $X'' + \lambda^2 X = 0$, whose general solution is

$$X(x) = c_1 \cos(\lambda x) + c_2 \sin(\lambda x) \quad ,$$

Rewriting the second equation as: $\frac{T'(t)}{T(t)} = -\lambda^2 - 3$ and again as

$$T'(t) + (\lambda^2+3)T(t) = 0 \quad ,$$

whose general solution is

$$T(t) = c_3 e^{-(\lambda^2+3)t} \quad .$$

So the product solution is:

$$u(x,t) = (c_1\cos(\lambda x) + c_2\sin(\lambda x))c_3 e^{-(\lambda^2+3)t} = C_1\cos(\lambda x)e^{-(\lambda^2+3)t} + C_2\sin(\lambda x)e^{-(\lambda^2+3)t} \quad .$$

We found two infinite families of product solutions

$$u(x,t) = \cos(\lambda x)e^{-(\lambda^2+3)t} \quad , \quad u(x,t) = \sin(\lambda x)e^{-(\lambda^2+3)t} \quad , \quad .$$

Now it is time to look at the boundary condition $u_x(0,t)=0$. For the second family, $u(x,t) = \sin(\lambda x)e^{-(\lambda^2+3)t}$,

$$u_x(x,t) = \lambda\cos(\lambda x)e^{-(\lambda^2+3)t} \quad , \quad .$$

So

$$u_x(0,t) = \lambda\cos(\lambda\cdot 0)e^{-(\lambda^2+3)t} = \cos(0)e^{-(\lambda^2+3)t} = e^{-(\lambda^2+3)t} \quad ,$$

that is **never** zero, so this family is out of the question!
For the first family, $u(x,t) = \cos(\lambda x)e^{-(\lambda^2+3)t}$,

$$u_x(x,t) = -\lambda\sin(\lambda x)e^{-(\lambda^2+3)t} \quad ,$$

So

$$u_x(0,t) = -\lambda\sin(\lambda\cdot 0)e^{-(\lambda^2+3)t} = 0 \quad .$$

So this family is OK! So far, λ can be *any* real number. Now it is time to look at the second boundary condition:

$$u_x(\pi,t) = 0$$

Getting

$$u_x(\pi,t) = -\lambda \sin(\lambda\cdot\pi)e^{-(\lambda^2+3)t} = 0 \quad .$$

Since $e^{-(\lambda^2+3)t}$ is not the zero function, we need

$$\sin(\lambda\cdot\pi) = 0 \quad ,$$

so we have to solve this **trig. eq.** Recall that the general solution of $\sin w = 0$ is $w = n\pi$ (n integer). So our λ must satisfy the relation

$$\lambda\pi = n\pi$$

Solving for λ we get $\lambda = n$ (n integer), so our previously infinite family of "building blocks" is still infinite, but much more restricted

$$u_n(x,t) = \cos(nx)e^{-(n^2+3)t} \quad n = 0,1,2,3,\ldots .$$

By the **principle of superposition** the following is the general solution of the pde plus the two boundary conditions

$$u(x,t) = \sum_{n=0}^{\infty} A_n \cos(nx)e^{-(n^2+3)t} \quad ,$$

for *any* numbers $A_0, A_1, A_2, \ldots$. Now it is time to look at the *initial condition* $u(x,0) = f(x)$,

$$u(x,0) = \sum_{n=0}^{\infty} A_n \cos(nx)e^{-(n^2+3)\cdot 0} = \sum_{n=0}^{\infty} A_n \cos(nx) \quad ,$$

But that's an old friend! It is the Fourier cosine half-range expansion of $f(x)$. But in this problem $f(x) = \cos 9x$, its Fourier-cosine expansion is **ITSELF!**, so $A_9 = 1$ and all the other A_n's are zero, and to get to $u(x,t)$ we simply stick-in $e^{-(n^2+3)t}$ with $n = 9$ in front of it!, in other words, e^{-84t}. So the **final answer** is simply $u(x,t) = (\cos 9x)e^{-84t}$.

Comments: This was probably the hardest problem! Some people confused it with the Heat Equation, and blindly used the canned formula for the Heat Equation. Wrong! This pde is similar to the Heat Equation, but not the same, so the canned formula is, of course, not applicable!

2. (15 points) Find the eigenvalues λ_n, and the corresponding eigenfunctions $y_n(x)$ for the following boundary value problem.

$$y'' + \lambda^2 y = 0 \quad , \quad y(0) = 0 \quad , \quad y(42) = 0 \quad .$$

Ans.: $\lambda_n = \frac{n\pi}{42}$, $y_n(x) = \sin(\frac{n\pi}{42}x)$.

Sol. The **general solution** of the ode is:

$$y(x) = c_1 \cos(\lambda x) + c_2 \sin(\lambda x) \quad .$$

Plugging-in $x = 0$, gives

$$y(0) = c_1 \cos(\lambda \cdot 0) + c_2 \sin(\lambda \cdot 0) = c_1$$

Since we are told that $y(0) = 0$, we get $c_1 = 0$, and the general solution shrinks to:

$$y(x) = c_2 \sin(\lambda x) \quad .$$

Now we plug-in $x = 42$:

$$y(42) = c_2 \sin(\lambda 42) \quad .$$

Since we don't want c_2 to be 0, we need

$$\sin(\lambda 42) \quad ,$$

so we have to solve this trig. equation. Recall that the solution of $\sin w = 0$ is $w = n\pi$ (n integer), so setting the *inside* of $\sin(\lambda 42)$, namely $\lambda 42$ equal to $n\pi$, we have to solve for λ:

$$\lambda 42 = n\pi \quad .$$

Dividing by 42 gives

$$\lambda = \frac{n\pi}{42} \quad (n \quad integer) \quad .$$

These are the **eigenvalues**, let's call them $\lambda_n = \frac{n\pi}{42}$. To get the corresponding **eigenfunctions** we simply plug-in $\lambda = \frac{n\pi}{42}$ into

$$y(x) = \sin(\lambda x) \quad ,$$

getting

$$y_n(x) = \sin(\frac{n\pi}{42}x) \quad .$$

Comment: Some people had the answer

$$\lambda = \frac{n\pi}{42} \quad , \quad y_n(x) = \sin(\lambda x)$$

While this is, strictly speaking correct, it is expected to plug-in the correct λ into $y(x)$ as I did above.

3. (15 points) Solve the pde

$$9u_{xx} = u_{tt} \quad , 0 < x < \pi \quad , \quad t > 0 \quad ,$$

subject to the **boundary-conditions**

$$u(0,t) = 0 \quad , \quad u(\pi, t) = 0 \quad , \quad t > 0 \quad ,$$

and the **initial conditions**

$$u(x,0) = 0 \quad , \quad u_t(x,0) = 11\sin x - 120\sin 5x \quad , \quad 0 < x < \pi \quad .$$

Ans.: $u(x,t) = \frac{11}{3}\sin x \sin 3t - 8\sin 5x \sin 15t$.

Sol. This is the wave equation with $a = 3$.
It is a piece of case if you use Dr. Z.'s amazing shortcut. Since both $u(x,0)$ (that happens to be 0 in this problem, and $u_t(x,0)$ are already in Fourier-Sine expansion, leave them alone, and to get $u(x,t)$ change each $\sin nx$ in the $u(x,0) = f(x)$ part into $\sin nx \cos(nat)$ (not applicable in this problem, since the $f(x)$ in the initial condition $u(x,0) = f(x)$ happens to be the zero function in this problem, and change each $\sin nx$ in $g(x)$ featuring in the initial condition $u_t(x,0) = g(x)$ into $\sin nx \sin(nat)/(na)$.
For the $11\sin x$ piece, $n = 1$ so we multiply it by $\sin(1 \cdot 3t)/3 = \sin(3t)/3$. For the $-120\sin 5x$ piece, $n = 5$ so we multiply it by $\sin(5 \cdot 3 \cdot t)/15 = \sin(5t)/15$, getting $-120\sin 5x \sin(5t)/15$. So the answer is

$$u(x,t) = \frac{11}{3}\sin x \sin 3t - 120\frac{\sin 5x \sin 15t}{8} = \frac{11}{3}\sin x \sin 3t - 15\sin 5x \sin 15t \quad .$$

4. (15 points) Find the half-range sine expansion of $f(x) = 3$ on $(0, 3\pi)$.

Ans.: $\frac{12}{\pi} \sum_{k=0}^{\infty} \frac{1}{2k+1} \sin(\frac{(2k+1)x}{3})$.

Note:: $\frac{6}{\pi} \sum_{n=1}^{\infty} \frac{1-(-1)^n}{n} \sin(\frac{nx}{3})$ is also an acceptable answer.

Sol. Since the interval is $(0, 3\pi)$ that is not the "nice" interval $(0, \pi)$ (i.e. $L = 3\pi$) we must first consider a brand-new function $g(x)$ whose natural habitat is $(0, \pi)$ defined by:

$$g(x) = f(\frac{L}{\pi}x)$$

the idea being that $g(\pi) = f(L)$. Here $L = 3\pi$, so

$$g(x) = f(\frac{3\pi}{\pi}x) = f(3x) \quad .$$

For future reference, once we would have the half-range sine expansion of $g(x)$, we will get back to $f(x)$ by

$$f(x) = g(\frac{x}{3}) \quad .$$

In this problem $f(x) = 3$ so $g(x) = 3$ (**NOT** 9!, like many people did). To get from $f(x)$ to $g(x)$ you replace x by $3x$, but $f(x)$ is a **constant** function!, it has no x in it, so there is nothing to do and $g(x) = 3$ also!
Now we can use the canned formula for the Fourier sine expansion

$$g(x) = \sum_{n=1}^{\infty} b_n \sin nx \quad ,$$

where the **coefficients**, b_n, are given by the integral formula

$$b_n = \frac{2}{\pi} \int_0^{\pi} g(x) \sin nx \, dx \quad .$$

In this problem, $g(x) = 3$ so

$$b_n = \frac{2}{\pi} \int_0^{\pi} 3 \sin nx \, dx = \frac{6}{\pi} \int_0^{\pi} \sin nx \, dx = \frac{6}{\pi} \left(\frac{-\cos nx}{n} \Big|_0^{\pi} \right) =$$

$$-\frac{6}{n\pi}\left(\cos nx\Big|_0^\pi\right) = -\frac{6}{n\pi}(\cos(n\pi) - \cos 0) = -\frac{6}{n\pi}((-1)^n - 1) = \frac{6}{n\pi}(1 - (-1)^n)$$

So

$$g(x) = \sum_{n=1}^{\infty} \frac{6}{n\pi}(1 - (-1)^n)\sin nx$$

Taking out the constant $\frac{6}{\pi}$ out of the $\sum$ (WARNING: SOME STUDENTS ALSO TAKE n stuff out! WRONG WRONG WRONG, this is NONSENSE!), we get

$$g(x) = \frac{6}{\pi}\sum_{n=1}^{\infty} \frac{1 - (-1)^n}{n}\sin nx \quad .$$

Finally, going back to $f(x)$, using $f(x) = g(x/3)$ we get

$$f(x) = \frac{6}{\pi}\sum_{n=1}^{\infty} \frac{1 - (-1)^n}{n}\sin\frac{nx}{3} \quad .$$

This is correct and *acceptable*. To get an even better answer, note that when n is an even integer $n = 2, 4, 6$, etc. $1 - (-1)^n$ is always 0 so it is not necessary to include them. When n is an odd integer $n = 1, 3, 5$, etc. $1 - (-1)^n$ is always 2. So writing a *typical* odd integer as $n = 2k + 1$ (but k now starts at 0), we can rewrite the above correct answer, even more efficiently, as:

$$f(x) = \frac{12}{\pi}\sum_{k=0}^{\infty} \frac{1}{2k+1}\sin\frac{(2k+1)x}{3} \quad .$$

5. (15 points altogether)
(a) (8 points) Show that the following set of two functions, over the given interval and weight function, is an orthogonal set.

$$\{\, f_1(x) = 1, \quad f_2(x) = 10x - 8 \,\} \qquad [0,1] \quad , \quad w(x) = x^3 \quad .$$

(b) (7 points) Using **orthogonality** (no credit for other methods!) find numbers $c1, c2$ such that

$$10x = c_1 f_1(x) + c_2 f_2(x) \quad .$$

Ans. to b): $c_1 = 8 \quad , \quad c_2 = 1$.

Sol. of a:

$$(f_1(x), f_2(x))_w = \int_0^1 f_1(x) f_2(x) x^3\, dx == \int_0^1 1(10x - 8)x^3\, dx = \int_0^1 (10x^4 - 8x^3)\, dx$$

$$= (10\frac{x^5}{5} - 8\frac{x^4}{4})\Big|_0^1 = (2x^5 - 2x^4)\Big|_0^1 = 0 - 0 = 0 \quad .$$

So $f_1(x)$ and $f_2(x)$ are indeed orthogonal with respect to the weight function $w(x) = x^3$.

Sol. o b:

$$c_1 = \frac{(f(x), f_1(x))_w}{(f_1(x), f_1(x))_w} = \frac{(10x, 1)_x^3}{(1,1)_{x^3}} \quad , \quad c_2 = \frac{(f(x), f_2(x))_w}{(f_2(x), f_2(x))_w} = \frac{(10x, 10x - 8)_{x^3}}{(10x - 8, 10x - 8))_{x^3}} \quad .$$

Let's do c_1 first: The numerator is:

$$(f(x), f_1(x))_{x^3} = \int_0^1 (10x)(1)x^3\, dx = \int_0^1 10x^4\, dx = 2x^5\Big|_0^1 = 2(1^5 - 0^5) = 2 \quad .$$

The denominator is:

$$(1,1)_{x^3} = \int_0^1 (1)(1)(x^3)\, dx = \frac{x^4}{4}\Big|_0^1 = \frac{1^1 - 0^4}{4} = \frac{1}{4} \quad .$$

So

$$c_1 = \frac{2}{\frac{1}{4}} = 8 \quad .$$

Regarding c_2, the numerator is

$$(10x, 10x - 8)_{x^3} = \int_0^1 (10x)(10x - 8)x^3 \, dx$$

while the denominator is

$$(10x-8, 10x-8)_{x^3} = \int_0^1 (10x-8)^2 x^3 \, dx = \int_0^1 (100x^2-160x+64)x^3 \, dx = \int_0^1 (100x^5-160x^4+64x^3) \, dx$$

$$= (100x^6/6 - 160x^5/5 + 64x^4/4)\Big|_0^1 = (50/3 - 32 + 16) = (50/3 - 16) = \frac{2}{3} \quad .$$

So

$$c_2 = \frac{2/3}{2/3} = 1 \quad .$$

6. (15 points) Solve :

$$u_{xx} + u_{yy} = 0 \quad , \quad 0 < x < \pi \quad , \quad 0 < y < 1 \quad ,$$

Subject to

$$u_x(0,y) = 0 \quad , \quad u_x(\pi,y) = 0 \quad , \quad 0 < y < 1 \quad ;$$

$$u_y(x,0) = 0 \quad , \quad u(x,1) = (\cosh 4)\cos 4x + (\cosh 7)\cos 7x + (\cosh 10)\cos 10x \quad , \quad 0 < x < \pi \quad .$$

Ans.: $u(x,y) = \cos 4x \cosh 4y + \cos 7x \cosh 7y + \cos 10x \cosh 10y$

Sol. There are **eight** infinite families of product solutions of the 2D Laplace equation. The four most common ones are

$$(i) \quad u(x,y) = \cos \lambda x \cosh \lambda y$$

$$(ii) \quad u(x,y) = \cos \lambda x \sinh \lambda y$$

$$(iii) \quad u(x,y) = \sin \lambda x \cosh \lambda y$$

$$(iv) \quad u(x,y) = \sin \lambda x \sinh \lambda y$$

and the four other ones

$$(v) \quad u(x,y) = \cosh \lambda x \cos \lambda y \quad , \quad (vi) \quad u(x,y) = \cosh \lambda x \sin \lambda y \quad ,$$

$$(vii) \quad u(x,y) = \sinh \lambda x \cos \lambda y \quad , \quad (viii) \quad u(x,y) = \sinh \lambda x \sin \lambda y \quad .$$

Now it is time to **eliminate** most of these options, using the conditions $u_x(0,y) = 0$, $u_y(x,0) = 0$.
For (i): $u(x,y) = \cos \lambda x \cosh \lambda y$

$$u_x(x,y) = -\lambda \sin \lambda x \cosh \lambda y \quad , \quad u_y(x,y) = \lambda \cos \lambda x \sinh \lambda y \quad ,$$

So $u_x(0,y) = 0$ and $u_y(x,0) = 0$ so (i) is still OK!
For (ii): $u(x,y) = \cos \lambda x \sinh \lambda y$

$$u_x(x,y) = -\lambda \sin \lambda x \sinh \lambda y \quad , \quad u_y(x,y) = \lambda \cos \lambda x \cosh \lambda y \quad ,$$

So $u_x(0,y) = 0$ (ok) and $u_y(x,0) = \lambda \cos \lambda x$ (**not** ok!) so (ii) is OUT!

For (iii): $u(x,y) = \sin \lambda x \cosh \lambda y$

$$u_x(x,y) = \lambda \cos \lambda x \cosh \lambda y \quad , \quad u_y(x,y) = \lambda \sin \lambda x \cosh \lambda y \quad ,$$

So $u_x(0,y) \neq 0$ (**not** ok) so (iii) is OUT!
For (iv): $u(x,y) = \sin \lambda x \sinh \lambda y$

$$u_x(x,y) = \lambda \cos \lambda x \sin \lambda y \quad ,$$

So $u_x(0,y) \neq 0$ (**not** ok) so (iv) is OUT!
Similarly, (vi),(vii), and (viii) are eliminated.
The only the infinite families that are still options are

$$\cos \lambda x \cosh \lambda y \quad , \quad \cosh \lambda x \cos \lambda y \quad , \quad .$$

Now we use $u_x(\pi, y) = 0$. With

$$u(x,y) = \cos \lambda x \cosh \lambda y \quad ,$$

we have

$$u_x(x,y) = -\lambda \sin \lambda x \cosh \lambda y \quad ,$$

Plugging-in $x = \pi$ gives

$$u_x(\pi,y) = -\lambda \sin(\lambda\pi) \cosh \lambda y \quad ,$$

Setting this equal to 0 yields the trig. equation

$$-\lambda \sin(\lambda\pi) \cosh \lambda y = 0$$

which is the same as

$$\sin(\lambda\pi) = 0 \quad .$$

The solution is $\lambda\pi = n\pi$ (n integer) giving $\lambda = n$, n integer.
For the other option

$$u(x,y) = \cosh \lambda x \cos \lambda y \quad ,$$

we would get

$$\sinh(\lambda\pi) = 0 \quad .$$

that has no solutions, so we can forget about this option. (Note, people who did not consider this option did not get penalized!)
So the **building block** solutions for the pde plus the first three boundary conditions are

$$u_n(x,y) = \cos nx \cosh ny \quad .$$

By the **principle of superposition** any linear combination, finite or infinite

$$u(x,y) = \sum_{n=1}^{\infty} A_n \cos nx \cosh ny$$

for *any* numbers $A_1, A_2, A_3 \ldots$ is yet another solution. It is time to impose the last boundary condition $u(x,1) = f(x)$ for

$$f(x) = (\cosh 4)\cos 4x + (\cosh 7)\cos 7x + (\cosh 10)\cos 10x \quad , \quad 0 < x < \pi \quad .$$

Plugging $y = 1$ into the general $u(x,y)$ above gives:

$$f(x) = \sum_{n=1}^{\infty} A_n \cos nx (\cosh n) = \sum_{n=1}^{\infty} (A_n \cosh n) \cos nx$$

This rings a bell! It is the Fourier-cosine expansion of $f(x)$. But our $f(x)$ is already in that form

$$f(x) = (\cosh 4)\cos 4x + (\cosh 7)\cos 7x + (\cosh 10)\cos 10x \quad , \quad 0 < x < \pi \quad .$$

Comparing coefficients

$$A_4 \cosh 4 = \cosh 4 \quad , \quad A_7 \cosh 7 = \cosh 7 \quad , \quad A_{10} \cosh 7 = \cosh 10 \quad .$$

and all the other A_n's are zero. So

$$A_4 = 1 \quad A_7 = 1 \quad A_{10} = 1 \quad ,$$

and all the remaining ones are 0, and going back to $u(x,y)$ we have

$$u(x,y) = \cos 4x \cosh 4y + \cos 7x \cosh 7y + \cos 10x \cosh 10y \quad .$$

7. (10 points) Find product solutions, if possible, to the partial differential equation

$$\frac{\partial u}{\partial x} - 5\frac{\partial u}{\partial y} = 0 \quad .$$

Ans.: $u(x,y) = e^{kx}e^{\frac{k}{5}y}$ or $u(x,y) = e^{kx+\frac{k}{5}y}$ or $u(x,y) = e^{k(5x+y)}$. (where k is *any* real number.

Sol. We first, write the **template**

$$u(x,y) = X(x)Y(y) \quad .$$

Since

$$u_x = X'(x)Y(y) \quad , \quad u_y = X(x)Y'(y) \quad .$$

Plugging it into the pde gives:

$$X'Y - 5XY' = 0 \quad .$$

As usual, dividing by XY:

$$\frac{X'Y - 5XY'}{XY} = 0 \quad .$$

Algebra:

$$\frac{X'Y}{XY} - \frac{5XY'}{XY} = 0 \quad .$$

More algebra:

$$\frac{X'}{X} - \frac{5Y'}{Y} = 0 \quad .$$

Moving the $Y(y)$ stuff to the right, and going back to *longhand*

$$\frac{X'(x)}{X(x)} = \frac{5Y'(y)}{Y(y)} \quad .$$

The left side **only** depends on x, the right side **only** depends on y, so the left side **does not** depend on y, the right side **does not depend** on x, but they are **equal**!. So **neither** side depends on x and neither side depends on y, so **both** sides are *just* a constant! Let's call that constant k, and we have now **two odes**:

$$\frac{X'(x)}{X(x)} = k \quad ,$$

$$\frac{5Y'(y)}{Y(y)} = k \quad .$$

In standard form:

$$X'(x) - kX(x) = 0 \quad ,$$

$$Y'(y) - (k/5)X(y) = 0$$

From calc4, the **general solutions** are

$$X(x) = c_1 e^{kx} \quad ,$$

$$Y(y) = c_2 e^{(k/5)y} \quad .$$

Going back to the **template**, we have

$$u(x,y) = c_1 c_2 e^{kx} e^{(k/5)y} = C e^{kx} e^{(k/5)y} \quad .$$

(c_1, c_2 are *arbitrary* constants, so we can rename $c_1 c_2$ as C, yet-another arbitrary constant. Since the equation is homogeneous, we get just put $C = 1$, and not mention it, since $u(x,y) = e^{kx}e^{(k/5)y}$ being a solution automatically implies that $u(x,y) = Ce^{kx}e^{(k/5)y}$ is a solution for any C, so it customary not to write the C (in other words, take $C = 1$), but it is not a mistake to leave the answer as $Ce^{kx}e^{(k/5)y}$ (C and k any numbers).