NAME: (print!) _____

E-Mail address: _____

MATH 421 (2), Dr. Z. , Exam 2, Tue., Nov. 26, 2024, 10:20-11:40am, SEC 217

No Calculators!, You can only use the official "cheatsheet" downloaded from
`http://www.math.rutgers.edu/~zeilberg/calc5_2014/cheatsheet.pdf` .
Write the final answer to each problem in the space provided. Incorrect answers (even due to minor errors) can receive at most one half partial credit, so please check and double-check your answers.

Do not write below this line (office use only)

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1. (out of 15)

2. (out of 15)

3. (out of 15)

4. (out of 15)

5. (out of 15)

6. (out of 15)

7. (out of 10)

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total (out of 100)

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1. (15 pts.) Solve (from scratch!) the boundary value problem

$$\frac{\partial^2 u}{\partial x^2} - 3u = \frac{\partial u}{\partial t} \quad , \quad 0 < x < \pi \quad , \quad t > 0 \quad ,$$

subject to

$$u_x(0,t) = 0 \quad , \quad u_x(\pi,t) = 0 \quad , t > 0$$

$$u(x,0) = \cos 9x \quad , \quad 0 < x < \pi \quad .$$

Ans.: $u(x,t) =$

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2. (15 points) Find the eigenvalues λ_n, and the corresponding eigenfunctions $y_n(x)$ for the following boundary value problem.

$$y'' + \lambda^2 y = 0 \quad , \quad y(0) = 0 \quad , \quad y(42) = 0 \quad .$$

Ans.: $\lambda_n =$ $y_n(x) =$

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3. (15 points) Solve the pde

$$9u_{xx} = u_{tt} \quad , 0 < x < \pi \quad , \quad t > 0 \quad ,$$

subject to the **boundary-conditions**

$$u(0,t) = 0 \quad , \quad u(\pi,t) = 0 \quad , \quad t > 0 \quad ,$$

and the **initial conditions**

$$u(x,0) = 0 \quad , \quad u_t(x,0) = 11\sin x - 120\sin 5x \quad , \quad 0 < x < \pi \quad .$$

Ans.: $u(x,t) =$

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4. (15 points) Find the half-range sine expansion of $f(x) = 3$ on $(0, 3\pi)$.

Ans.:

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5. (15 points altogether)
(a) (8 points) Show that the following set of two functions, over the given interval and weight function, is an orthogonal set.

$$\{\, f_1(x) = 1, \quad f_2(x) = 10x - 8 \,\} \qquad [0,1] \quad , \quad w(x) = x^3 \quad .$$

(b) (7 points) Using **orthogonality** (no credit for other methods!) find numbers $c1, c2$ such that

$$10x = c_1 f_1(x) + c_2 f_2(x) \quad .$$

Ans. to b): $c_1 =$ $c_2 =$

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6. (15 points) Solve :

$$u_{xx} + u_{yy} = 0 \quad , \quad 0 < x < \pi \quad , \quad 0 < y < 1 \quad ,$$

Subject to

$$u_x(0, y) = 0 \quad , \quad u_x(\pi, y) = 0 \quad , \quad 0 < y < 1 \quad ;$$

$$u_y(x, 0) = 0 \quad , \quad u(x, 1) = (\cosh 4) \cos 4x + (\cosh 7) \cos 7x + (\cosh 10) \cos 10x \quad , \quad 0 < x < \pi \quad .$$

Ans.: $u(x, y) =$

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7. (10 points) Find product solutions, if possible, to the partial differential equation

$$\frac{\partial u}{\partial x} - 5\frac{\partial u}{\partial y} = 0 \quad .$$

Ans.: $u(x, y) =$
