NAME: (print!) ____

E-Mail address: ____

MATH 421 (2), Dr. Z. , FINAL EXAM, Monday, Dec. 23, 2024, 8:00-11:00pm, SEC 117

No Calculators!, You can only use the official "cheatsheet" downloaded from
http://www.math.rutgers.edu/~zeilberg/calc5/cheatsheet.pdf
Write the final answer to each problem in the space provided. Incorrect answers (even due to minor errors) can receive at most one half partial credit, so please check and double-check your answers.
I am a member (please circle) of Only Scc1 / Only Scc2 / Both Scc1 and Scc2 / None

Do not write below this line (office use only)

---

1. (out of 15)

2. (out of 15)

3. (out of 15)

4. (out of 15)

5. (out of 15)

6. (out of 15)

7. (out of 15)

8. (out of 15)

9. (out of 15)

10. (out of 15)

11. (out of 15)

12. (out of 15)

13. (out of 10)

14. (out of 10)

----

total (out of 200)

----

1. (15 pts.) Solve (from scratch!) the boundary value problem

$$\frac{\partial^2 u}{\partial x^2} + 6u = 3\frac{\partial u}{\partial t} \quad , \quad 0 < x < \pi \quad , \quad t > 0 \quad ,$$

subject to

$$u(0,t) = 0 \quad , \quad u(\pi,t) = 0 \quad , t > 0$$
$$u(x,0) = \sin 3x \quad , \quad 0 < x < \pi \quad .$$

Ans.: $u(x,t) =$

---

2. (15 pts.) Find the eigenvalues λ_n, and the corresponding eigenfunctions $y_n(x)$ for the following boundary value problem.

$$y'' + \lambda^2 y = 0 \quad , \quad y'(0) = 0 \quad , \quad y'(10) = 0 \quad .$$

Ans.: $\lambda_n =$ $y_n(x) =$

---

3. (15 pts.) Solve the pde

$$25u_{xx} = u_{tt} \quad ,0 < x < \pi \quad , \quad t > 0 \quad ,$$

subject to the **boundary-conditions**

$$u(0,t) = 0 \quad , \quad u(\pi,t) = 0 \quad , \quad t > 0 \quad ,$$

and the **initial conditions**

$$u(x,0) = \sin 4x \quad , \quad u_t(x,0) = 5\sin x + 10\sin 5x \quad , \quad 0 < x < \pi \quad .$$

Ans.: $u(x,t) =$

---

4. (15 pts.) Find the half-range cosine expansion of $f(x) = x$ on $(0, 2\pi)$.

Ans.:

---

5. (15 pts. altogether)
(a) (8 points) Show that the following set of two functions, over the given interval and weight function, is an orthogonal set.

$$\{ f_1(x) = 1, \quad f_2(x) = 5x - 3 \} \qquad [0, 1] \quad , \quad w(x) = \sqrt{x} \quad .$$

(b) (7 points) Using **orthogonality** (no credit for other methods!) find numbers c_1 , c_2 such that

$$5x = c_1 f_1(x) + c_2 f_2(x) \quad .$$

Ans. to b): $c_1 =$ $\qquad$ $c_2 =$

---

6. (15 pts.) Solve :

$$u_{xx} + u_{yy} = 0 \quad , \quad 0 < x < \pi \quad , \quad 0 < y < 1 \quad ,$$

Subject to

$$u(0, y) = 0 \quad , \quad u(\pi, y) = 0 \quad , \quad 0 < y < 1 \quad ;$$

$$u(x, 0) = 0 \quad , \quad u(x, 1) = (\sinh 4) \sin 4x + (\sinh 7) \sin 7x + (\sinh 10) \sin 10x \quad , \quad 0 < x < \pi \quad .$$

Ans.: $u(x, y) =$

---

7. (15 pts.) Find product solutions, if possible, to the partial differential equation

$$2\frac{\partial u}{\partial x} + 3\frac{\partial u}{\partial y} = 0 \quad .$$

Ans.: $u(x, y) =$

---

8. (15 pts.) Find

$$\mathcal{L}^{-1}\left\{ \frac{3s^2-1}{s^3-s} \right\}$$

Ans.:

---

9. (15 pts.) **9a.** (7 points) Compute $\mathcal{L}\{(t+6)\mathcal{U}(t-6)\}$.

Ans.:

9b. (8 points) Compute

$$\mathcal{L}^{-1}\{\frac{e^{-4s}}{(s+2)^3}\} \quad .$$

Ans.:

---

10. (15 pts.) Evaluate

$$\mathcal{L}\{\int_0^t \tau^{15} e^{3t-3\tau}\, d\tau\} \quad .$$

Ans.:

---

11. (15 pts.) Solve the initial-value problem

$$y'' + 6y' + 9y = \delta(t-1) \quad , \quad y(0) = 0 \quad , \quad y'(0) = 0 \quad .$$

Ans.:

12.(15 pts.) Using the Laplace Transform (no credit for other methods) solve the pde

$$u_{xx} = 4u_{tt} \quad , \quad 0 < x < \pi \quad , \quad t > 0$$

subject to the **boundary-conditions**

$$u(0,t) = 0 \quad , \quad u(\pi,t) = 0 \quad , \quad t > 0 \quad ,$$

and the initial conditions

$$u(x,0) = \sin(3x) \quad , \quad u_t(x,0) = 0 \quad , \quad 0 < x < \pi \quad ,$$

Ans.:

---

13. (10 points) Approximate, with mesh-size $h = 1$, the solution of the boundary-value problem

$$u_{xx} + u_{yy} = 0 \quad , \quad 0 < x < 2 \quad , \quad 0 < y < 2 \quad ;$$

subject to the boundary conditions

$$u(0,y) = 1 \quad , \quad 0 < y < 2 \quad ; \quad u(2,y) = 4 \quad , \quad 0 < y < 2 \quad ;$$

$$u(x,0) = 2 \quad , \quad 0 < x < 2 \quad ; \quad u(x,2) = -2 \quad , \quad 0 < x < 2 \quad .$$

Ans.: The approximation of $u(1,1)$ is:

---

14. (10 pts.) Find all the eigenvalues of the matrix

$$\begin{bmatrix} 10 & -6 \\ 12 & -7 \end{bmatrix} \quad ,$$

and determine a basis for each eigenspace.

Ans.: smaller eigenvalue: corresponding eigenfunction:

larger eigenvalue: corresponding eigenfunction:
