**Solutions to Dr. Z.'s Math 421(2), (Fall 2014, RU) REAL Quiz #3**

**1.** Compute (using Lecture 4's method!) $\mathcal{L}\{t^3 e^{-2t}\}$ .

**Sol. to 1**: We are supposed to use the formula

$$\mathcal{L}\{t^n f(t)\} = (-1)^n F^{(n)}(s) \quad ,$$

where $F(s) = \mathcal{L}\{f(t)\}$.

Here $f(t) = e^{-2t}$ so $F(s) = \frac{1}{s+2} = (s+2)^{-1}$. We have $F'(s) = (-1)(s+2)^{-2}$, $F''(s) = (-1)(-2)(s+2)^{-3}$, and $F'''(s) = (-1)(-2)(-3)(s+2)^{-4} = -\frac{6}{(s+2)^4}$ Hence

$$\mathcal{L}\{t^3 e^{-2t}\} = \frac{6}{(s+2)^4} \quad .$$

**Ans. to 1**: $\frac{6}{(s+2)^4}$.

**2.** Solve the IVP

$$y' - 3y = \delta(t-2) \quad , \quad y(0) = 1 \quad .$$

**Sol. to 2**: Appply $\mathcal{L}$, we get

$$\mathcal{L}\{y'\} - 3\mathcal{L}\{y\} = \{\delta(t-2)\} = e^{-2s}$$

Putting, as usual $\mathcal{L}\{y\} = Y$, we have

$$sY - y(0) - 3Y = e^{-2s}$$

But, since $y(0) = 1$, we have

$$sY - 1 - 3Y = e^{-2s}$$

Solving for $Y$:

$$(s-3)Y = 1 + e^{-2s} \quad ,$$

dividing by $(s-3)$:

$$Y = \frac{1}{s-3} + \frac{e^{-2s}}{s-3} \quad .$$

Since $\mathcal{L}^{-1}\{\frac{1}{s-3}\} = e^{3t}$, we get,

**Ans. to 2**:

$$y(t) = e^{3t} + e^{3(t-2)}\mathcal{U}(t-2) \quad .$$