NAME: (print!) _____

E-Mail address: _____

MATH 421 (2), Dr. Z. , Exam 2, Tue., Nov. 25, 2014, 12:00noon-1:20pm, SEC 216

No Calculators!, You can only use the official "cheatsheet" downloaded from
`http://www.math.rutgers.edu/~zeilberg/calc5_2014/cheatsheet.pdf` .
Write the final answer to each problem in the space provided. Incorrect answers (even due to minor errors) can receive at most one half partial credit, so please check and double-check your answers.

Do not write below this line (office use only)

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1. (out of 15)

2. (out of 15)

3. (out of 15)

4. (out of 15)

5. (out of 15)

6. (out of 15)

7. (out of 10)

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total (out of 100)

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1. (15 pts.) Find the general expression, in polar coordinates, for the steady-state temperature $u(r, \theta)$ in a circular plate of radius 3, if the temperature on the circumference $r = 3$ is given by $u(3, \theta) = \cos\theta - 2\sin 3\theta$

Ans.: $u(r, \theta) =$

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2. (15 points altogether) (a) (8 points) Show that $\{\sin nx\}$, $n = 1, 2, \ldots$ is orthogonal over the interval $[0, \pi]$.
(b) Find the norm of each function.

Ans. to (b): $||\sin nx|| =$

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3. (15 points) Find the **complex** Fourier series of the function $f(x) = x$ on the interval $-\pi < x < \pi$.

Ans.:

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4. (15 points) The **Hermite** polynomials $H_n(x)$ are defined by

$$H_n(x) = 2xH_{n-1}(x) - 2(n-1)H_{n-2}(x)$$

with initial conditions $H_0(x) = 1, H_1(x) = 2x$. Find $H_4(x)$.

Ans.: $H_4(x) =$

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5. (15 points) Solve the boundary value pde problem:

$$4u_{xx} = u_{tt} \quad , 0 < x < \pi \quad , \quad t > 0 \quad ;$$

$$u(0,t) = 0 \quad , \quad u(\pi,t) = 0 \quad , \quad t > 0 \quad ;$$

$$u(x,0) = \sin 3x + \sin 4x \quad , \quad u_t(x,0) = \sin 2x - \sin 5x \quad , \quad 0 < x < \pi \quad .$$

Ans: $u(x,t) =$

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6. (15 points) Find the Fourier series of $f(x) = e^{2x} + 5$ over the interval $(-\pi, \pi)$.

Ans.:

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7. (10 points) Set up the boundary value problem for the steady-state temperature $u(x, y)$, where a thin rectangular plate coincides with the region in the xy-plane defined by $0 \leq x \leq 30, 0 \leq y \leq 40$ if The bottom, left and right sides are insulated, while the top side is held at temperature e^x.

Ans.: pde:

boundary conditions:
