

Dr. Z.'s Calc5 Homework assignment 8

1: Find the Fourier series of

$$f(x) = \begin{cases} 0, & \text{if } -\pi < x < 0; \\ 1, & \text{if } 0 \leq x < \pi. \end{cases}$$

2: Find the Fourier series of

$$f(x) = \begin{cases} 1, & \text{if } -3\pi < x < 0; \\ 2, & \text{if } 0 \leq x < 3\pi. \end{cases}$$

3: Find the Fourier series of $f(x) = |x|$ on $(-\pi, \pi)$. In other words:

$$f(x) = \begin{cases} -x, & \text{if } -\pi < x < 0; \\ x, & \text{if } 0 \leq x < \pi. \end{cases}$$

4: Find the Fourier series of $f(x) = x^2$ on the interval $(-\pi, \pi)$.

Note: You may use the ready-made indefinite integrals:

$$\begin{aligned} \int x^2 \sin nx &= \frac{(2 - n^2 x^2) \cos(nx) + 2nx \sin(nx)}{n^3} \\ \int x^2 \cos nx &= \frac{(n^2 x^2 - 2) \sin(nx) + 2xn \cos(nx)}{n^3} . \end{aligned}$$

5. Find the Fourier series of $f(x) = e^x$ over the interval $(-\pi, \pi)$.

Note: You may use the ready-made indefinite integrals:

$$\begin{aligned} \int e^x \sin nx &= \frac{(\sin nx - n \cos nx)e^x}{n^2 + 1} \\ \int e^x \cos nx &= \frac{(\cos nx + n \sin nx)e^x}{n^2 + 1} \end{aligned}$$