

Dr. Z.'s Calc5 Homework assignment 7

Version of Oct. 13, 2014

1. Show that the given functions are orthogonal on the given interval.

$$f_1(x) = e^{x^2} x^5 \quad , \quad f_2(x) = e^{-x^2} (x^4 + x^2) \quad , \quad [-1, 1] \quad .$$

2. Determine whether the following set of functions constitute an orthogonal set on the interval $[0, 1]$.

$$f_1(x) = 1 \quad , \quad f_2(x) = 2x - 1 \quad , \quad f_3(x) = 3x^2 - 1 \quad .$$

3. Determine whether the following set of functions constitute an orthogonal set on the interval $[0, 1]$ with respect to the weight function $w(x) = x^2$.

$$f_1(x) = 1 \quad , \quad f_2(x) = 4x - 9 \quad .$$

4. Show that $\{\sin nx\}$, $n = 1, 2, \dots$ is orthogonal over the interval $[0, \pi]$. Also find the norm of each function.

5. Show that $\{\cos(n\pi/3)x\}$, $n = 0, 1, 2, \dots$ is orthogonal over the interval $[0, 3]$. Also find the norm of each function.

6. Verify by direct integration that the functions are orthogonal with respect to the indicated weight function on the given interval

$$L_0(x) = 1 \quad , \quad L_1(x) = -x + 1 \quad , \quad L_2(x) = \frac{1}{2}x^2 - 2x + 1 \quad ; \quad w(x) = e^{-x} \quad , \quad [0, \infty) \quad .$$

7. [YOU ARE ENCOURAGED TO USE MAPLE]

(a) Show that the following set of functions, over the given interval and weight function is an orthogonal set.

$$\{f_1(x) = 1, \quad f_2(x) = 5x - 4 \quad , \quad f_3(x) = 21x^2 - 30x + 10\}, \quad [0, 1] \quad , \quad w(x) = x^3 \quad .$$

(b) Using **orthogonality** (no credit for other methods!) find numbers c_1, c_2, c_3 such that

$$21x^2 = c_1 f_1(x) + c_2 f_2(x) + c_3 f_3(x) \quad .$$

(c) Check directly the answer to part (b).