

Dr. Z.'s Calc5 Homework assignment 15

1. Solve

$$\frac{\partial^2 u}{\partial x^2} = \frac{\partial u}{\partial t} \quad , \quad 0 < x < 2\pi \quad , \quad t > 0$$

subject to the conditions

$$u(0, t) = 0 \quad , \quad u(2\pi, t) = 0 \quad , \quad t > 0$$

$$u(x, 0) = x^2(2\pi - x)$$

(You may use the ready-made formula) **Hint:**

$$\int_0^{2\pi} x^2(2\pi - x) \sin(nx/2) dx = \frac{-32\pi(1 + 2(-1)^n)}{n^3} \quad .$$

2. Solve

$$3 \frac{\partial^2 u}{\partial x^2} = \frac{\partial u}{\partial t} \quad , \quad 0 < x < 2\pi \quad , \quad t > 0$$

subject to the conditions

$$u(0, t) = 0 \quad , \quad u(2\pi, t) = 0 \quad , \quad t > 0$$

$$u(x, 0) = 300$$

(You may use the ready-made formula)

3. Solve the boundary value problem

$$7 \frac{\partial^2 u}{\partial x^2} = \frac{\partial u}{\partial t} \quad , \quad 0 < x < \pi \quad , \quad t > 0 \quad ,$$

subject to

$$u_x(0, t) = 0 \quad , \quad u_x(\pi, t) = 0 \quad , \quad t > 0$$

$$u(x, 0) = f(x) \quad , \quad 0 < x < \pi \quad ,$$

where

$$f(x) = \begin{cases} 0, & \text{if } 0 \leq x < \pi/2; \\ 2, & \text{if } \pi/2 \leq x < \pi; \end{cases}$$

(You may use the ready-made formula)

4. Solve (from scratch!) the boundary value problem

$$\frac{\partial^2 u}{\partial x^2} - u = \frac{\partial u}{\partial t} \quad , \quad 0 < x < \pi \quad , \quad t > 0 \quad ,$$

subject to

$$u_x(0, t) = 0 \quad , \quad u_x(\pi, t) = 0 \quad , \quad t > 0$$

$$u(x, 0) = f(x) \quad , \quad 0 < x < \pi \quad ,$$