1. (10 points) Find the general expression, in polar coordinates, for the steady-state temperature $u(r, \theta)$ in the infinite plane with a circular hole of radius 2 cut-out, and where the temperature at the bounding ring is $u(2, \theta) = \cos 2\theta - 3\sin 4\theta, 0 < \theta < 2\pi$.

Solution. The boundary function $f(\theta) = u(2, \theta) = \cos 2\theta - 3\sin 4\theta$ is its own Fourier Series, so all we need is multiply each $\sin n\theta$ and/or $\cos n\theta$ by $(r/c)^{-n}$ (since this is an infinite plate, we need negative powers).

Here $c = 2$. There is one pure-cosine term, and there is one pure-sine term.

For $\cos 2\theta$ $n = 2$ so we multiply it by $(r/2)^{-2}$, getting $(r/2)^{-2}\cos 2\theta = 4r^{-2}\cos 2\theta$.

For $-3\sin 4\theta$ $n = 4$ so we multiply it by $(r/2)^{-4}$, getting $-3\sin 4\theta(r/2)^{-4} = -3(\sin 4\theta) \cdot (16r^{-4}) = -48r^{-4}\sin 4\theta$

so the answer is simply

$$u(r, \theta) = 4r^{-2}\cos 2\theta - 48r^{-4}\sin 4\theta$$

Comment: Many people didn’t get it. What a shame! It is such an easy problem. Please review the problem and make sure you understand how to do it.