1. Use the Laplace Transform to solve the given initial-value problem.

\[ y'' - 3y' + 2y = 0 \quad , \quad y(0) = 2 \quad , \quad y'(0) = 3 \]

**Sol.:** First apply \( \mathcal{L} \):

\[
\mathcal{L}\{y'' - 3y' + 2y\} = 0 .
\]

Now \( \mathcal{L}\{y''\} = s^2Y - sy(0) - y'(0) = s^2Y - 2s - 3 \), \( \mathcal{L}\{y'\} = sY - y(0) = sY - 2 \), to get

\[
\mathcal{L}\{y''\} - 3\mathcal{L}\{y'\} + 2\mathcal{L}\{y\} = s^2Y - 2s - 3 - 3(sY - 2) + 2Y = 0
\]

Expanding:

\[
s^2Y - 2s - 3 - 3sY + 6 + 2Y = s^2Y - 2s + 3 - 3sY + 2Y = 0 .
\]

Collecting terms, and moving the non-\( Y \) stuff to the right side:

\[
(s^2 - 3s + 2)Y = 2s - 3 \quad .
\]

Solving for \( Y \), we get

\[
Y = \frac{2s - 3}{s^2 - 3s + 2} .
\]

We now need to apply \( \mathcal{L}^{-1} \), but first we need to express \( \frac{2s-3}{s^2-3s+2} \) in terms of partial-fractions. Factorizing

\[
Y = \frac{2s-3}{(s-1)(s-2)} .
\]

Using the template \( \frac{A}{s-1} + \frac{B}{s-2} \) we get:

\[
Y = \frac{2s-3}{(s-1)(s-2)} = \frac{A}{s-1} + \frac{B}{s-2} = \frac{A(s-2) + B(s-1)}{(s-1)(s-2)} .
\]

The denominators match automatically, so we set the numerators equal to each other

\[
2s - 3 = A(s - 2) + B(s - 1) .
\]

Convenient values: \( s = 2 \) gives \( 2(2) - 3 = A(0) + B(2 - 1) \) so \( B = 1 \). \( s = 1 \) gives \( 2(1) - 3 = A(1 - 2) + B(0) \), so \(-1 = -A \) and \( A = 1 \). Going back to the template:

\[
Y = \frac{1}{s-1} + \frac{1}{s-2} .
\]

Now it is time to apply \( \mathcal{L}^{-1} \):

\[
y(t) = \mathcal{L}^{-1}\{\frac{1}{s-1} + \frac{1}{s-2}\} = e^t + e^{2t} .
\]

**Ans.:** \( y(t) = e^t + e^{2t} \).

**Comment:** About \%40 of the students scored the full 10 points. Another \%30 knew how to do it, but made algebraic or numerical errors.