1. Using the definition find the Laplace transform $L\{f(t)\}$ (alias $F(s)$) of $f(t) = e^{-t} + 3$.

Sol.

$$L\{f(t)\} = \int_0^\infty e^{-st}(e^{-t} + 3) \, dt = \int_0^\infty e^{-st}e^{-t} \, dt + \int_0^\infty e^{-st} \cdot 3 \, dt = \int_0^\infty e^{-st-t} \, dt + 3 \int_0^\infty e^{-st} \, dt =$$

$$\int_0^\infty e^{-(s+1)t} \, dt + 3 \int_0^\infty e^{-st} \, dt = \frac{e^{-\infty}}{-(s+1)} - \frac{e^0}{-(s+1)} + 3 \frac{e^{-\infty}}{-s} - 3 \frac{e^0}{-s} = \frac{1}{s+1} + \frac{3}{s} .$$

Ans. to 1.: $\frac{1}{s+1} + \frac{3}{s}$.

Comment: About 65% got it completely right. Some people had trouble plugging in and doing the algebra.

2. Using Tables, find $L\{f(t)\}$, if $f(t) = (t+1)(t-1) + 3t$.

Sol.

$$L\{f(t)\} = L\{(t+1)(t-1) + 3t\} = L\{t^2 - 1 + 3t\} = L\{t^2 + 3t - 1\} = \frac{2!}{s^3} + \frac{3!}{s^2} - \frac{1}{s} = \frac{2}{s^3} + \frac{3}{s^2} - \frac{1}{s} .$$

Ans. to 2.: $F(s) = \frac{2}{s^3} + \frac{3}{s^2} - \frac{1}{s}$.

Comment. About 85% of the students got it completely right. Some lost a point or two by not simplifying $2!$ to 2, or messing up the very simple high-school algebra.