1. Differentiate the following functions
   \[ a. \ f(x) = xe^x + 6 \quad b. \ g(x) = x \cos 5x \quad c. \ h(x) = \sin 2x + 5 \cos 4x \]

   **Sol. to 1a:**
   \[ f'(x) = (xe^x + 6)' = x'e^x + xe^x = e^x + xe^x = (x + 1)e^x \]

   **Sol. to 1b:**
   \[ g'(x) = x' \cos 5x + x(\cos 5x)' = \cos 5x - 5x \sin 5x \]

   **Sol. to 1c:**
   \[ h'(x) = 2 \cos 2x + 5(- \sin 4x)(4) = 2 \cos 2x - 20 \sin 4x \]

   **Comments:** Most people got it completely right. The rest only messed up a little.

2. Perform the following definite integrations:
   \[ a. \ \int_0^\ln 3 e^{2x} \, dx \quad b. \ \int_0^\pi x \sin 2x \, dx \]

   **Sol. to 2a:**
   \[ \int_0^\ln 3 e^{2x} \, dx = \left. \frac{e^{2x}}{2} \right|_0^\ln 3 = \frac{e^{2 \ln 3} - e^0}{2} = \frac{9 - 1}{2} = 4 \]

   [Most people got so far, and that’s correct, but only %50 managed to simplify further, using the algebra of ln. Those students who forgot how to simplify ln expressions are very welcome to come to free tutoring ARC 206, Thurs. Sept. 8, 2011, 8:40-10:00 am]

   Continuing
   \[ \frac{e^{2 \ln 3} - e^0}{2} = \frac{e^{\ln 9} - e^0}{2} = \frac{9 - 1}{2} = \frac{8}{2} = 4 \]

   **Final Ans. to 2a:** 4.

   **Sol. to 2b:** Integration by parts:
   \[ \int uv' \, dx = uv - \int u'v \, dx \]

   We choose
   \[ u = x \quad , \quad v' = \sin 2x \]

   (the other choice \( u = \sin 2x \), \( v' = x \) would give us an even more complicated integral!). So

   \[ u' = 1 \quad , \quad v = -\frac{1}{2} \cos 2x \]

   and
   \[ \int x \sin 2x \, dx = (x)(-\frac{1}{2} \cos 2x) - \int 1 \cdot (-\frac{1}{2} \cos 2x) \, dx = \]
\[- \frac{x \cos 2x}{2} - \frac{1}{2} \int \cos 2x \, dx = - \frac{x \cos 2x}{2} + \frac{1}{4} \sin 2x \, .\]

Putting the limits of integration, we get:

\[
\int_{0}^{\pi} x \sin 2x \, dx = - \frac{x \cos 2x}{2} \bigg|_{0}^{\pi} + \frac{1}{4} \sin 2x \bigg|_{0}^{\pi} = 0 - \frac{\pi \cos 2\pi}{2} + 0 - 0 = - \frac{\pi}{2} \, .
\]

Ans. to 2b: \(- \frac{\pi}{2}\).

Comment: about half of the people got it right. Another quarter were on the right track, the rest either didn’t do it at all, or messed up completely.

3. Find the function \(y(x)\) if

\[y' - 2y = 0 \quad , \quad y(0) = 3\, .\]

Sol. to 3: We have to solve

\[
\frac{dy}{dx} = 2y
\]

By separation of variables:

\[
\frac{dy}{y} = 2 \, dx \, .
\]

Now integrate both sides:

\[
\int \frac{dy}{y} = \int 2 \, dx \, .
\]

and get

\[
\ln y = 2x + C \, .
\]

Now exponentiate:

\[
e^{\ln y} = e^{2x+C} = e^{2x} e^{C} \, .
\]

Watch out: quite a few people, did: \(e^{2x+C} = e^{2x} + e^{C}\), this is WRONG!). Now we rename \(e^{C}\) to be \(C\) and use \(e^{\ln y} = y\) (from the algebra of \(\ln\)) to get the General solution

\[
y(x) = Ce^{2x} \, .
\]

Now it is time to use the initial condition: \(y(0) = 3\).

\[
y(0) = Ce^{2 \cdot 0} = Ce^{0} = C \, .
\]

So \(C = 3\). Going back to the general solution, we have:

\[
y(x) = 3e^{2x} \, .
\]

Comment: Only about \%50 of the students did it completely right. It is also possible to do it using the method of calc4, as in problem 4 below.
4. Find the function $y(x)$ if

$$y'' - 3y' + 2y = 0 \quad , \quad y(0) = 2 \quad , \quad y'(0) = 3 \quad .$$

**Sol. of 4:** The characteristic equation (using $r$) is

$$r^2 - 3r + 2 = 0$$

Factorizing:

$$(r - 1)(r - 2) = 0 \quad ,$$

so we get two solutions $r = 1, r = 2$, so $e^x$ and $e^{2x}$ are both solutions, and since the ode is **homogeneous**, the **general solution** is

$$y(x) = Ae^x + Be^{2x} \quad ,$$

where $A$ and $B$ are **arbitrary constants**.

Now it is time to take care of the **initial conditions**. For future reference:

$$y'(x) = Ae^x + 2Be^{2x} \quad .$$

We have, upon plugging-in $x = 0$:

$$y(0) = Ae^0 + Be^{2\cdot 0} = A + B$$

$$y'(0) = Ae^0 + 2Be^{2\cdot 0} = A + 2B \quad .$$

But $y(0) = 2, y'(0) = 3$, so we have the **system** of two linear equations with two unknowns, $A$ and $B$:

$$A + B = 2 \quad , \quad A + 2B = 3 \quad .$$

Solving, we get $A = 1, B = 1$. Going back to the general solution, we get

$$y(x) = 1 \cdot e^x + 1 \cdot e^{2x} = e^x + e^{2x} \quad .$$

**Ans. to 4:** $y(x) = e^x + e^{2x}$.

**Comment:** Most people were on the right track, but quite a few people messed up the simple algebra at the end. About half of the students got the right answer.

5. If $f(x, y) = \sin xy$, find the partial derivatives $f_x(x, y)$ and $f_y(x, y)$.

**Sol. of 5:**

$$f_x(x, y) = y \cos xy \quad , \quad f_y(x, y) = x \cos xy \quad .$$

**Comment:** These are **very basic** calc3 problems. %80 of the students got them right, but the remaining %20 students got them wrong. Those people are encouraged to come to free tutoring ARC 206, 8:40am, every Monday, and this coming Thurs., Sept. 8, 2011.