Solutions to the Attendance Quiz for Dec. 8, 2011

1.: Use the improved Euler method to find an approximate value for $y(3)$ if $y(x)$ is the solution of the initial value problem ode

$$ y' = 2x - 3y + 1 \quad , \quad y(1) = 3 \quad . $$

Using the mesh-size $h = 1$.

**Sol.** From the cheatsheet

$$ y^*_n = y_{n-1} + hf(x_{n-1}, y_{n-1}) \quad . $$

$$ y_n = y_{n-1} + \frac{hf(x_{n-1}, y_{n-1}) + f(x_n, y^*_n)}{2} $$

In this problem, $h = 1, x_0 = 1, y_0 = 3$. So $x_1 = 2, x_2 = 3$. We need $y_2$, but first we need $y_1$.

$$ y^*_1 = y_0 + (1) f(x_0, y_0)) = 3 + (1) f(1, 3) = 3 + (2(1) - 3(3) + 1) = 3 + 2 - 9 + 1 = -3 \quad . $$

Now that we have $y^*_1$ we are ready for $y_1$:

$$ y_1 = y_0 + h \frac{f(x_0, y_0) + f(x_1, y_1^*)}{2} = 3 + (1) \frac{f(1, 3) + f(2, -3)}{2} = 3 + (1) \frac{2(1) - 3(3) + 1 + (2(2) - 3(-3) + 1)}{2} $$

$$ = 3 + \frac{2 - 9 + 1 + 4 + 9 + 1}{2} = 3 + \frac{8}{2} = 7 \quad . $$

Next we need $y^*_2$:

$$ y^*_2 = y_1 + (1) f(x_1, y_1)) = 7 + (1) f(2, 7) = 7 + (2(2) - 3(7) + 1) = 7 + 4 - 21 + 1 = -9 \quad . $$

Finally, to get $y_2$, we have

$$ y_2 = y_1 + h \frac{f(x_1, y_1) + f(x_2, y_2^*)}{2} = 7 + (1) \frac{f(2, 7) + f(3, -9)}{2} = 7 + \frac{2(2) - 3(7) + 1 + (2(3) - 3(-9) + 1)}{2} $$

$$ = 7 + \frac{4 - 21 + 1 + 6 + 27 + 1}{2} = 7 + \frac{18}{2} = 7 + 9 = 16 \quad . $$

**Ans.:** $y(3)$ is approximately equal to 16.

**Comments:** I didn’t let you have enough time, so only about 20% of the people finished the problem correctly. Many got as far as $y_1$ correctly, but then messed up the calculations. Please review this kind of problems. Once you know how to do them, they are fairly straightforward, and it would be a pity to mess them up in the Final exam. Remember, for an incorrect answer, you can get at most half partial credit.