Dr. Z.’s Calc5 Homework assignment 1

1. Use the definition of the Laplace Transform to compute $\mathcal{L}\{f(t)\}$ if
   
   $$f(t) = t + 2$$

2. Use the definition of the Laplace Transform to compute $\mathcal{L}\{f(t)\}$ if
   
   $$f(t) = 4e^t + 2e^{-t}$$

3. Use the definition of the Laplace Transform to compute $\mathcal{L}\{f(t)\}$ if
   
   $$f(t) = \begin{cases} 2, & \text{if } 0 \leq t \leq 3; \\ -2, & \text{if } t \geq 3. \end{cases}$$

4. Prove, by induction on $k$, that
   
   $$\mathcal{L}\{t^k\} = \frac{k!}{s^{k+1}} \quad (k = 0, 1, 2, 3, \ldots),$$

   **Hint:** The statement for $k$ is
   
   $$\int_0^\infty t^k e^{-st} \, dt = \frac{k!}{s^{k+1}}$$

   Now differentiate both sides with respect to $s$, and get the analogous statement for $k + 1$.

In 5-9 use *any* method to compute $\mathcal{L}\{f(t)\}$ if

5. $f(t) = t^3 + 2t + 11$.
6. $f(t) = (t + 1)(t - 2)$.
7. $f(t) = e^t + 2e^{3t} + t^3$.
8. $f(t) = (e^t + 1)(e^t + 2)$.
9. $f(t) = 2e^t - 3\sin t + 2\cos 5t$. 