Answers to Dr. Z.'s Math 421 (Calc5) Homework assignments (when applicable)

Disclaimer: not responsible for any errors. The first finder of any error will get $1.

Version of Dec. 12, 2011: (found by Chris Crossley): Changing correcting 21#2 ($e^{-\alpha^2 t}$ corrected to $e^{-(\alpha^2/4)t}$

Previous corrections: Sept. 10, 2011, 3:10pm: Ryan Godfrey (who won a dollar, correcting the last term of #9 in 1.) Sept. 14, 2011: Matthew Dominguez (who won a dollar, correcting #3 in 1. that had a typo); Sept. 15, 2011: correcting a typo in #10 of 3. (a factor of s was missing before, found by Dr. Z. himself); Sept. 17, 2011:correcting a typo in #2 of 3. (found by Ryan Godfrey who won another dollar). Sept. 19, 2011, 9:05pm: correcting typos in #1 and #2 of 6. (found by Dr. Z.). Sept. 29, 2011: correcting #2 of the Linear Algebra assignment, thanks to Pratik Desai who won a dollar. (The previous version refereed to a 3×3 matrix that was in the original version of that assignment. To make it easier, I changed it to a 2×2 matrix, but forgot to change the answer.) Oct. 12, 2011: (found by Modupe Sonaike). In assignment 7, changing 6(b) to 7(b). Oct. 16, 2011: (found by Ming Tai Ha): correcting the second part of #1 of assignment 8. Oct. 17, 2011: (found by Josh Vichare): correcting the a_0 part of #5 of assignment 8. (previously there was an extra factor of π at the bottom). Oct. 18, 2011: (found by Tom Hansen): correcting the cosine part of #5 of assignment 8. (previously there was an extra factor of n in the numerator of the cosine part). Oct. 24, 2011: (found by Dat Nguyen): correcting #3 in 10 (previously there was a factor of i missing) and # 5 in 10 ($\frac{1}{2}$ was missing and a factor of $\frac{1}{\pi}$ in front of the $\sum$. #5' changed accordingly. (found by Eric Leung): correctiong #3 of 12 (the last term of $Z_4(x)$ was previously incorrect). Oct. 24, 2011: (found by Eric Leung): adding yet-another solution to #4 of 13 (the case when $k = 0$, yielding the simple solution $y(x) = c_1 x + c_2$. Oct. 31, 2011 (found by Billy): adding more stuff to #5 of hw 13. Oct. 31, 2011: (found by Joseph Ross): correcting the signs in #4 of hw 12. Nov. 8, 2011: (found by Joey Reichert): correcting #2a of assignment 14 Nov. 10, 2011: (found by Dr. Z.): correcting #3 of assignment 18. Version of Nov. 18, 2011:(found by Chris Farina): correcting #5' of assignment 10 (remove $(n \neq 0)$) Nov. 19, 2011: (found by Ryan Godfrey): correcting #1 of assignment 18 ($r^n \cos 7\theta$ changed to the correct $r^7 \cos 7\theta$). Version of Nov. 20, 2011: (found by Joseph Ross): correcting #4 of assignment 18 (in the sine part, it is $(2k+1)$ in the denominator **not** $(2k+1)^2$). Dec. 1, 2011: (found by Ying Wang): correcting the A_0 term in of assignment 18 #5. Dec. 6, 2011: (found by Ryan Godfrey): correcting the numerators in assignment 20 #4. Dec. 6, 9:30am, 2011: (found by Matthew Richmond): correcting the numerators in assignment 21 #4 Dec. 7 2011: (found by Tom Hansen): Changing U to U_x in one of the terms of assignment 19 #3.

Assignment 1: 1. $\frac{1}{s^2} + 2\frac{1}{s}$ 2. $\frac{4}{s-1} + \frac{2}{s+1}$ 3. $\frac{2(1-2e^{-3s})}{s}$ 5. $\frac{6}{s^4} + \frac{2}{s^2} + \frac{11}{s}$ 6. $\frac{2}{s^3} - \frac{1}{s^2} - \frac{2}{s}$ 7. $\frac{1}{s-1} + \frac{2}{s-3} + \frac{6}{s^4}$ 8. $\frac{2}{s} + \frac{1}{s-2} + \frac{3}{s-1}$ 9. $\frac{2}{s-1} - \frac{3}{s^2+1} + \frac{2s}{s^2+25}$.

Assignment 2: 1. $\frac{t^3}{6}$ 2. $t^2 - 6t$ 3. $1 + 2t + \frac{1}{2}t^2$ 4. $\frac{1}{4}e^t - \frac{1}{4}e^{-3t}$ 5. $\frac{1}{5} - \frac{1}{5}\cos(\sqrt{5}t)$ 6. $\frac{1}{3}e^{-2t} - \frac{1}{2}e^{-t} + \frac{1}{6}e^t$ 7. $y(t) = -\frac{1}{2} + \frac{1}{2}e^{2t}$ 8. $y(t) = 2e^{2t} + e^{-2t}$ 9. $y(t) = e^{2t} + e^t$ 10. $y(t) = -e^{-t} + e^t + e^{2t}$ 11. $y(t) = \frac{2}{3}\sin t - \frac{1}{3}\sin 2t$.

Assignment 3: 1. $\frac{120}{(s+2)^6}$ 2. $\frac{2}{(s-3)^2+4}$ 3. $\frac{s-4}{(s-4)^2+4} - \frac{2}{(s-2)^3}$ 4. $\frac{1}{3}e^t \sin 3t$ 5. $e^{3t}(2\cos t + 7\sin t)$ 6. $y(t) = (t-1)e^t$ 7. $y(t) = \frac{1}{12}t^4 e^t$ 8. $(t+1) - (t+1)\mathcal{U}(t-3)$ 9. $e^{-3s}(\frac{9}{s} + \frac{6}{s^2} + \frac{2}{s^3})$ 10. $\frac{se^{-(\pi/2)s}}{s^2+1}$ 11. $\frac{1}{6}(t-2)^3\mathcal{U}(t-2)$ 12. $(t-3)e^{t-3}\mathcal{U}(t-3)$ 13. $(e^{2t}-1) - \frac{3}{2}(e^{2(t-1)}-1)\mathcal{U}(t-1)$.

Assignment 4: 1. $\frac{4s}{(s^2+4)^2}$ 2. $\frac{2}{(s-2)^3}$ 3. $\frac{6}{(s+1)^4}$ 4. $\frac{1}{s^2(s-1}$ 5. $\frac{1}{s^2+1}$ 6. $f(t) = -\frac{1}{8}e^{-t} + \frac{1}{8}e^t(2t^2+6t+1)$ 7. $f(t) = e^{-t}$ 8. $y(t) = \frac{1}{2}t\cos t + \frac{1}{2}(t-1)\sin t$.

Assignment 5: 1. $y(t) = -e^{2t} + e^{2t-6}\mathcal{U}(t-3)$ 2. $y(t) = -e^{-t} + e^{-t+1}\mathcal{U}(t-1) + 2e^{-t+2}\mathcal{U}(t-2) + 3e^{-t+3}\mathcal{U}(t-3)$ 3. $y(t) = \frac{1}{2}\sin 2t\,\mathcal{U}(t-\pi)$ 4. $y(t) = (t-2)e^{-2t+4}\mathcal{U}(t-2)$ 5. $y(t) = -e^{-t+\pi}\sin t\,\mathcal{U}(t-\pi)$.

Assignment 6: 1. $x(t) = \frac{1}{3}e^t - \frac{1}{3}e^{-2t}$, $y(t) = \frac{2}{3}e^t + \frac{1}{3}e^{-2t}$ 2. $x(t) = -\frac{7}{3}\sin 3t + 2\cos 3t$, $y(t) = -\frac{5}{3}\sin 3t - \cos 3t$ 3. $x(t) = -e^{-t} + \frac{1}{2}t^2 + t + 3$, $y(t) = \frac{t+1}{3}e^{-t} - \frac{1}{3}$ 4. $x(t) = e^t + e^{2t}$, $y(t) = e^t - e^{2t}$.

Assignment for Linear Algebra Refresher: 1. $1 : \{\begin{bmatrix}3\\2\end{bmatrix}\}$; $2 : \{\begin{bmatrix}2\\1\end{bmatrix}\}$. 2. $1 : \{\begin{bmatrix}1\\-1\end{bmatrix}\}$; $-1 : \{\begin{bmatrix}1\\-2\end{bmatrix}\}$.

Assignment 7: 2. No 3. No 4. norm: $\sqrt{\frac{\pi}{2}}$ 5. norm: $\sqrt{\frac{3}{2}}$ 7.(b) $c_1 = 14, c_2 = 6, c_3 = 1$.

Assignment 8: 1. $\frac{1}{2} + \sum_{n=1}^{\infty} \frac{1-(-1)^n}{n\pi}\sin nx$, or better: $\frac{1}{2} + \frac{2}{\pi}\sum_{n=0}^{\infty}\frac{1}{(2n+1)}\sin(2n+1)x$. 2. $\frac{3}{2} - \frac{2}{\pi}\sum_{n=0}^{\infty}\frac{1}{2n+1}\sin(\frac{(2n+1)x}{3})$ 3. $\frac{\pi}{2} - \frac{4}{\pi}\sum_{n=0}^{\infty}\frac{\cos(2n+1)x}{(2n+1)^2}$ 4. $\frac{\pi^2}{3} + \sum_{n=1}^{\infty}\frac{4(-1)^n}{n^2}\cos nx$ 5.

$$\frac{e^\pi - e^{-\pi}}{2\pi} + \frac{e^\pi - e^{-\pi}}{\pi}\sum_{n=1}^{\infty}\frac{(-1)^n}{n^2+1}\cos nx - \frac{e^\pi - e^{-\pi}}{\pi}\sum_{n=1}^{\infty}\frac{(-1)^n n}{n^2+1}\sin nx \quad .$$

Assignment 9: 1. odd,even, neither, even, odd, neither. 2. $\frac{\pi}{2} - \frac{2}{\pi}\sum_{n=0}^{\infty}\frac{1-(-1)^n}{n^2}\cos nx$ or better: $\frac{\pi}{2} - \frac{4}{\pi}\sum_{n=0}^{\infty}\frac{1}{(2n+1)^2}\cos(2n+1)x$ 3. $\frac{1}{2} - \frac{2}{\pi^2}\sum_{n=0}^{\infty}\frac{1-(-1)^n}{n^2}\cos(n\pi x)$ or better: $\frac{1}{2} - \frac{4}{\pi^2}\sum_{n=0}^{\infty}\frac{1}{(2n+1)^2}\cos((2n+1)\pi x)$ 4. $\frac{3}{2} + \frac{2}{\pi}\sum_{n=0}^{\infty}\frac{(-1)^n}{2n+1}\cos(2n+1)x$ 4'. $\frac{3}{2} + \frac{2}{\pi}\sum_{n=0}^{\infty}\frac{(-1)^n}{2n+1}\cos\frac{(2n+1)x\pi}{8}$ 5. $\sum_{n=1}^{\infty}\frac{8(-1)^{n+1}}{n}\sin\frac{nx}{4}$ 6. $2\pi - \frac{16}{\pi}\sum_{n=0}^{\infty}\frac{1}{(2n+1)^2}\cos\frac{(2n+1)x}{4}$.

Assignment 10: 1. $\sum_{n=-\infty, n\neq 0}^{\infty}(-1)^n\frac{i}{n}e^{inx}$ 2. $\sum_{n=-\infty, n\neq 0}^{\infty}(-1)^n\frac{2i}{n}e^{inx/2}$ 3. $\sum_{n=-\infty, n\neq 0}^{\infty} i(-1)^n\frac{n^2\pi^2-6}{n^3}e^{inx}$ 4. $64i\sum_{n=-\infty, n\neq 0}^{\infty}(-1)^n\frac{n^2\pi^2-6}{n^3}e^{inx/4}$ 5. $\frac{1}{2} - \frac{i}{\pi}\sum_{n=-\infty, n\neq 0}^{\infty}\frac{1}{2n+1}e^{(2n+1)ix}$ 1'. $\{(n, \frac{1}{|n|})\}$ $(n \neq 0)$

and $(0,0)$ 2'. $(n/2, \frac{2}{|n|})$ $(n \neq 0)$ and $(0,0)$ 3'. $(n, |\frac{n^2\pi^2-6}{n^3}|)$ $(n \neq 0)$ and $(0,0)$ 4'. $(n/4, 64|\frac{n^2\pi^2-6}{n^3}|)$ $(n \neq 0)$ and $(0,0)$ 5'. $(2n+1, 1/|2n+1|)$ (n integer) and $(0, \frac{1}{2})$.

Assignment 11: 1. $\{\sin \lambda_n x\}$, $\tan \lambda_n = -2\lambda_n$ 2. $\{\cos \lambda_n x\}$, $\cot 2\lambda_n = -3\lambda_n$ 3. $\{\cos(n + \frac{1}{2})x\}$, $n + \frac{1}{2}$ (n integer) 4. $\{\sin(\frac{n}{2})x\}$, $\frac{n}{2}$ (n integer) 5. $\{\sin \lambda_n x\}$, $\tan 10\lambda_n = -5\lambda_n$

Assignment 12: 1. $2x^2 - 1, 4x^3 - 3x, 8x^4 - 8x^2 + 1$ 2. $\frac{1}{2}x^2 - 2x + 1, -\frac{1}{6}x^3 + \frac{3}{2}x^2 - 3x + 1, \frac{1}{24}x^4 - \frac{2}{3}x^3 + 3x^2 - 4x + 1$ 3. $27x^4 - 4x + 9, 1728x^6 - 256x^3 + 571x^2 + 16x$ 4. $-\frac{1}{2}, \frac{3}{4}, 0, -\frac{7}{16}$ 5. $\frac{3}{5}P_1(x) + \frac{2}{5}P_3(x)$ 6. $\frac{e}{2} - \frac{1}{2e}, \frac{3}{e}, \frac{5e}{2} - \frac{35}{2e}$.

Assignment 13: 1. $e^{5kx}e^{-ky}$ 2. $e^{kx/11}e^{ky/5}$ 3. $x^k y^k$ 4. $e^{kx}e^{k^2y/9}$, $e^{-kx}e^{k^2y/9}$, $c_1x + c_2$, $e^{-k^2y/9}(\cos kx)$, $e^{-k^2y/9}(\sin kx)$ 5. $e^{\sqrt{k+1}x}e^{kt/9}$, $e^{-\sqrt{k+1}x}e^{kt/9}$ (when $k > -1$) $\cos \sqrt{-(k+1)}xe^{kt/9}$, $\sin \sqrt{-(k+1)}xe^{kt/9}$, (when $k < -1$), $e^{-t/9}$ (when $k = -1$). 6. impossible 7. hyperbolic 8. hyperbolic.

Assignment 14: (Warning: Do not peek at the answers before you do them yourself! In these problems, the answers and solutions coincide. Just copying the answers will not do you any good!)

1.a: $ku_{xx} = u_t, 0 < x < 10, 0 < t < \infty$; $u(0,t) = 1000, u_x(10,t) = 0; u(x,0) = 100$.

1.b: $ku_{xx} = u_t, 0 < x < 10, 0 < t < \infty$; $u(0,t) = 130, u_x(10,t) = 0; u(x,0) = x^2$.

1.c: $ku_{xx} = u_t, 0 < x < 10, 0 < t < \infty$; $u_x(0,t) = 0, u_x(10,t) = 0; u(x,0) = \sin x$.

2.a: $\alpha^2 u_{xx} = u_{tt}, 0 < x < 100, 0 < t < \infty; u(0,t) = 0, u(100,t) = 0 (0 \leq t < \infty); u(x,0) = 0, u_t(x,0) = \sin 20x (0 < x < 100)$

2.b: $\alpha^2 u_{xx} = u_{tt}, 0 < x < 100, 0 < t < \infty; u(0,t) = \sin 5t, u(100,t) = \cos 3t (0 \leq t < \infty); u(x,0) = x^3, u_t(x,0) = \cos 5x (0 < x < 100)$.

3.a: $u_{xx} + u_{yy} = 0 (0 < x < 10, 0 < y < 20); u(x,0) = 150, u(x,20) = 50 (0 < x < 10); u(0,y) = 150, u(10,y) = g(y) (0 < y < 20)$

3.b: $u_{xx} + u_{yy} = 0 (0 < x < 10, 0 < y < 20); u_y(x,0) = 0, u(x,20) = f(x) (0 < x < 10); u_x(0,y) = 0, u_x(10,y) = 0 (0 < y < 20)$

3.c: $u_{xx} + u_{yy} = 0 (0 < x < 10, 0 < y < 20); u(x,0) = 50, u(x,20) = 50 (0 < x < 10); u(0,y) = \sin 2y, u_x(10,y) = 0 (0 < y < 20)$.

Assignment 15: 1. $-32 \sum_{n=1}^{\infty} \frac{1+2(-1)^n}{n^3} e^{-n^2t/4} \sin(nx/2)$ 2. $-\frac{1200}{\pi} \sum_{k=0}^{\infty} \frac{1}{2k+1} e^{-3(2k+1)^2t/4} \sin((2k+1)x/2)$ 3. $1 + \frac{4}{\pi} \sum_{k=0}^{\infty} \frac{(-1)^k}{2k+1} e^{-7(2k+1)^2t} \cos((2k+1)x)$ 4. $u(x,t) = \frac{A_0}{2} + \sum_{n=1}^{\infty} A_n e^{-(n^2+1)t} \cos nx$, where $A_0 = \frac{2}{\pi} \int_0^{\pi} f(x)\,dx$, $A_n = \frac{2}{\pi} \int_0^{\pi} f(x) \cos nx\,dx$.

Assignment 16: 1. $u(x,t) = \cos 3t \sin 3x + \frac{1}{4}\sin 4t \sin 4x$ 2. $u(x,t) = 10\cos t \sin x + \frac{2}{3}\sin 3t \sin 3x$ 3. $u(x,t) = -4\sum_{n=1}^{\infty} \frac{1+2(-1)^n}{n^3}\cos nt \sin nx$ 4. $u(x,t) = 4\sum_{n=1}^{\infty} \frac{2+(-1)^n}{n^4}\sin nt \sin nx$ 5. $u(x,t) = -\frac{72}{\pi}\sum_{k=0}^{\infty} \frac{1}{(2k+1)^3}\cos(2k+1)t \sin\frac{(2k+1)x}{3}$.

Assignment 17: 1. $u(x,y) = \sum_{n=1}^{\infty} A_n \sin nx \sinh ny$, where $A_n = \frac{2}{\pi \sinh n}\int_0^{\pi} f(x)\sin nx\, dx$

2. $u(x,y) = \sum_{n=1}^{\infty} A_n \sin nx \cosh ny$, where $A_n = \frac{2}{\pi \cosh n}\int_0^{\pi} f(x)\sin nx\, dx$

3. $u(x,y) = A_0 + \sum_{n=1}^{\infty} A_n \cos nx \cosh ny$, where $A_0 = \frac{1}{\pi}\int_0^{\pi} f(x)\, dx \quad , \quad A_n = \frac{2}{\pi \cosh n}\int_0^{\pi} f(x)\cos nx\, dx$
.

4. $u(x,y) = u_1(x,y) + u_2(x,y)$, where $u_1(x,y)$ is a solution of:

$$(u_1)_{xx} + (u_1)_{yy} = 0 \quad , \quad 0 < x < 3 \quad , \quad 0 < y < 4$$

$$u_1(0,y) = y^3 \quad , \quad u_1(3,y) = \cos 7y \quad , \quad 0 < y < 4 \quad .$$

$$u_1(x,0) = 0 \quad , \quad u_1(x,4) = 0 \quad , \quad 0 < x < 3 \quad .$$

and $u_2(x,y)$ is a solution of:

$$(u_2)_{xx} + (u_2)_{yy} = 0 \quad , \quad 0 < x < 3 \quad , \quad 0 < y < 4$$

$$u_2(0,y) = 0 \quad , \quad u_2(3,y) = 0 \quad , \quad 0 < y < 4 \quad .$$

$$u_2(x,0) = x^3 \quad , \quad u_2(x,4) = e^x \quad , \quad 0 < x < 3 \quad .$$

Assignment 18: 1. $r^7 \cos 7\theta$. 2. $\frac{3}{32}(\cos 5\theta)r^5 + \frac{11}{8}(\sin 3\theta)r^3 - \frac{19}{2^{29}}(\sin 29\theta)r^{29}$
3. $50 + \frac{200}{\pi}\sum_{k=0}^{\infty} \frac{r^{2k+1}\sin(2k+1)\theta}{(2k+1)2^{2k+1}}$.
4.

$$-\frac{4}{\pi}\sum_{k=0}^{\infty} \frac{r^{2k+1}}{(2k+1)^2 3^{2k+1}}\cos(2k+1)\theta + 2\sum_{k=0}^{\infty} \frac{r^{2k+1}}{(2k+1)3^{2k+1}}\sin(2k+1)\theta$$

5.

$$u(r,\theta) = A_0 \quad + \sum_{n=1}^{\infty}(A_n \cos(n\theta) + B_n \sin(n\theta))r^{-n}$$

where

$$A_0 = \frac{1}{2\pi}\int_0^{2\pi} f(\theta) \quad , \quad A_n = \frac{c^n}{\pi}\int_0^{2\pi} f(\theta)\cos n\theta\, d\theta \quad , \quad B_n = \frac{c^n}{\pi}\int_0^{2\pi} f(\theta)\sin n\theta\, d\theta \quad .$$

Assignment 19: 1. $19U_{xxx} - 7U_{xx} = (s^2 + 4s - 11)U - (s+4)u(x,0) - u_t(x,0)$ 2. $29U_{xxx} - 17U_{xx} = (s^3 + 4s^2 - 11s + 5)U - (s^2 + 4s - 11)u(x,0) - (s+4)u_t(x,0) - u_{tt}(x,0)$ 3. $29U_{xx} +$

$11s^2U + 17sU_x - 17u_x(x,0) - 11su(x,0) - 11u_t(x,0)$ 4. $u(x,t) = 3\sin x \sin t - \frac{5}{4}\sin 4t \sin 4x$ 5. $(\frac{3}{8}\sin t - \frac{1}{8}\sin 3t)\sin x$ 6. $(2\sin t - \sin 2t)\sin 2x$

Assignment 20:

1.
$$\frac{1}{\pi}\int_0^\infty \frac{\sin\alpha}{\alpha}\cos\alpha x\, d\alpha - \frac{3}{\pi}\int_0^\infty \frac{\cos\alpha - 1}{\alpha}\sin\alpha x\, d\alpha \quad .$$

2.
$$\frac{1}{\pi}\int_0^\infty \frac{\cos 2\alpha + 2\alpha\sin 2\alpha - 1}{\alpha^2}\cos\alpha x\, d\alpha + \frac{1}{\pi}\int_0^\infty \frac{\sin 2\alpha - 2\alpha\cos 2\alpha}{\alpha^2}\sin\alpha x\, d\alpha \quad .$$

3.
$$\frac{1}{\pi}\int_0^\infty \frac{\alpha^2\sin\alpha - 2\sin\alpha + 2\alpha\cos\alpha}{\alpha^3}\cos\alpha x\, d\alpha + \frac{1}{\pi}\int_0^\infty \frac{2\alpha\sin\alpha - (\alpha^2-2)\cos\alpha - 2}{\alpha^3}\sin\alpha x\, d\alpha \quad .$$

4.
$$\frac{1}{\pi}\int_0^\infty \frac{1}{\alpha^2+1}\cos\alpha x\, d\alpha + \frac{1}{\pi}\int_0^\infty \frac{\alpha}{\alpha^2+1}\sin\alpha x\, d\alpha$$

5.
$$\frac{4}{\pi}\int_0^\infty \frac{1-3\alpha^2}{(\alpha^2+1)^3}\cos\alpha x\, d\alpha \quad , \quad \frac{4}{\pi}\int_0^\infty \frac{(3-\alpha^2)\alpha}{(\alpha^2+1)^3}\sin\alpha x\, d\alpha$$

6.
$$\frac{4}{\pi}\int_0^\infty \frac{1}{\alpha^2+4}\cos\alpha x\, d\alpha \quad , \quad \frac{2}{\pi}\int_0^\infty \frac{\alpha}{\alpha^2+4}\sin\alpha x\, d\alpha$$

Assignment 21: 1.
$$\frac{1}{\pi}\int_{-\infty}^\infty \frac{e^{-\alpha^2 t}\cos\alpha x}{\alpha^2+1}\, d\alpha$$

2.
$$\frac{10}{\pi}\int_{-\infty}^\infty \frac{(\cos\alpha - 1)e^{-\alpha^2 t/4}\sin(\alpha x)}{\alpha}\, d\alpha$$

3.
$$\frac{2}{\pi}\int_0^\infty \frac{\sinh(2\alpha x)\cos(\alpha y)}{(\alpha^2+1)\sinh(4\alpha)}\, d\alpha$$

4.
$$\frac{4}{\pi}\int_0^\infty \frac{\alpha\sinh(\alpha x)\sin(\alpha y)}{(\alpha^2+1)^2\sinh(2\alpha)}\, d\alpha$$

Assignment 22: 1. 26 2. $\frac{577}{8}$ 3. 1.452 4. 1.5179106 5. 1.521952697

Assignment 23: 1. $u(1,1) \approx \frac{5}{4}$ 2. $u(2,2) \approx \frac{15}{4}$ 3. $u(\frac{1}{2},\frac{1}{2}) \approx \frac{19}{24}, u(\frac{1}{2},1) \approx \frac{29}{24}, u(1,\frac{1}{2}) \approx \frac{23}{24}, u(1,1) \approx \frac{37}{24}$

4.

$$u_{2,3} + u_{3,2} - 4\,u_{3,3} = 3/2 \quad ,$$

$$u_{3,2} + u_{2,1} - 4\,u_{3,1} = -3/4 \quad ,$$

$$u_{2,1} + u_{1,2} - 4\,u_{1,1} = -5/4 \quad ,$$

$$u_{2,3} + u_{1,2} - 4\,u_{1,3} = -1/2$$

$$u_{3,3} + u_{2,2} + u_{3,1} - 4\,u_{3,2} = 1/2 \quad ,$$

$$u_{2,2} + u_{1,3} + u_{1,1} - 4\,u_{1,2} = -1/2 \quad ,$$

$$u_{3,1} + u_{2,2} + u_{1,1} - 4\,u_{2,1} = -1 \quad ,$$

$$u_{3,2} + u_{2,3} + u_{1,2} + u_{2,1} - 4\,u_{2,2} = 0 \quad ,$$

$$u_{3,3} + u_{1,3} + u_{2,2} - 4\,u_{2,3} = 1/2 \quad .$$