Solutions to the Attendance Quiz # 17 for Dr. Z.'s Calc4 for Lecture 17

1. Convert the following diff. eq. to a system of first-order diff.eqs

$$y^{(4)}(t) = \sqrt{(2y'''(t) + t^2 + y'(t)y(t)} + \cos y(t) + e^{y''(t)}$$

Sol. to 1: We define

$$x_1(t) = y(t) \quad , \quad x_2(t) = y'(t) \quad , \quad x_3(t) = y''(t) \quad , \quad x_4(t) = y'''(t) \quad , \quad .$$

The **first three equations** are easy! (and always the same!)

$$x_1'(t) = x_2(t)$$

$$x_2'(t) = x_3(t)$$

$$x_3'(t) = x_4(t)$$

The last equation is

$$x_4'(t) = \sqrt{(2x_4(t) + t^2 + x_2(t)x_1(t)} + \cos x_1(t) + e^{x_3(t)} \quad .$$

(Obtained by replacing $y(t)$ by $x_1(t)$, $y'(t)$ by $x_2(t)$, $y''(t)$ by $x_3(t)$, $y'''(t)$ by $x_4(t)$.)

Comment: Many people put $x_5(t)$ on the left hand side. This is **wrong**, there are only **four** functions in the system, since the given diff.eq. was **fourth order**. The left side is $x_4'(t)$.

2. Solve the initial value problem for the system, by using techniques for solving a single diff.eq. of higher order

$$x_1'(t) = 2x_2(t) \quad , \quad x_2'(t) = -8x_1(t) \quad ; \quad x_1(0) = 2 \quad , \quad x_2(0) = 32 \quad .$$

Sol. of 2: From the first equation we have

$$x_2(t) = \frac{1}{2}x_1'(t) \quad .$$

Plugging into the second

$$(\frac{1}{2}x_1'(t))' = -8x_1(t) \quad .$$

So

$$x_1''(t) = -16x_1(t) \quad ,$$

and moving everything to the left:

$$x_1''(t) + 16x_1(t) = 0 \quad .$$

Let's solve this diff. eq. The characteristic equation is

$$r^2 + 16 = 0 \quad .$$

So

$$r^2 = -16$$

and $r = 0 \pm 4i$. So the general solution is

$$x_1(t) = c_1 \cos 4t + c_2 \sin 4t \quad .$$

We need **initial conditions**. Of course, $x_1(0) = 2$, but what is $x_1'(0)$?. Going back to:

$$x_2(t) = \frac{1}{2} x_1'(t) \quad .$$

and plugging-in $t = 0$:

$$x_2(0) = \frac{1}{2} x_1'(0) \quad .$$

So $x_1'(0) = 2x_2(0) = 2 \cdot 32 = 64$.

Plugging-in $t = 0$ into the general solution gives:

$$x_1(0) = c_1 \cos 0 + c_2 0 = c_1 \quad .$$

So $c_1 = 2$. Differentiating the general solution we have

$$x_1'(t) = -4c_1 \sin 4t + 4c_2 \cos 4t \quad .$$

So, when $t = 0$:

$$x_1'(0) = 4c_2 \quad ,$$

since $x_1'(0) = 64$ we get

$$64 = 4c_2 \quad ,$$

so $c_2 = 16$.

Going back to the general solution, we have

$$x_1(t) = 2 \cos 4t + 16 \sin 4t \quad .$$

Finally, going back to

$$x_2(t) = \frac{1}{2} x_1'(t) \quad .$$

We have

$$x_2(t) = \frac{1}{2}(2 \cos 4t + 16 \sin 4t)' = \frac{1}{2}(-8 \sin 4t + 64 \cos 4t) = -4 \sin 4t + 32 \cos 4t \quad .$$

Ans. to 2:

$$x_1(t) = 2 \cos 4t + 16 \sin 4t \quad , \quad x_2(t) = -4 \sin 4t + 32 \cos 4t \quad .$$

Comments: Most people ran out of time (it was a very LONG problem), Congratulations to the few people who got it completely!