NAME: (print!) _____ **Section** _____

E-Mail address: _____

MATH 244 (1-3), Dr. Z. , Exam II, Thurs. Dec. 1, 2016, 8:40-10:00am, SEC 117

No Calculators! No Cheatsheets!
Write the final answer to each problem in the space provided. CHECK YOUR ANSWERS. SEE GRADING POLICY BELOW.

Do not write below this line (office use only)

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1. (out of 10)

2. (out of 10)

3. (out of 10)

4. (out of 10)

5. (out of 10)

6. (out of 10)

7. (out of 10)

8. (out of 10)

9. (out of 10)

10. (out of 10)

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total: (out of 100)

Reminders about Stability of Critical Points:
Two real and distinct eigenvalues **BOTH** positive, ($r_1 > r_2 > 0$): Node ; Unstable; Two real and distinct eigenvalues **BOTH** negative, ($r_1 < r_2 < 0$):Node ; Asymptotically Stable Two real and distinct eigenvalues of **OPPOSITE SIGN** ($r_2 < 0 < r_1$):Saddle Point ; Unstable
Repeated eigenvalue that is positive ($r_1 = r_2 > 0$):Proper or Improper Node ; Unstable (Note: if the eigenspace is two-dimensional, it is a proper node, if it is one-dimensional, it is improper)
Repeated eigenvalue that is negative ($r_1 = r_2 < 0$): Proper or Improper Node ; Asymptotically Stable (Note: if the eigenspace is two-dimensional, it is a proper node, if it is one-dimensional, it is improper)
Complex eigenvectors ($r_1, r_2 = \lambda \pm i\mu$) with positive real part (i.e. $\lambda > 0$): Spiral point ; Unstable Complex eigenvectors ($r_1, r_2 = \lambda \pm i\mu$) with negative real part, (i.e. $\lambda < 0$):Spiral point ; Asymptotically Stable
Complex eigenvectors ($r_1, r_2 = \lambda \pm i\mu$) with zero real part, (i.e. $\lambda = 0$):Center ; Stable

Reminder about the method of variation of parameters: If the functions $p(t), q(t), g(t)$ are continuous on an open interval I, and if $y_1(t)$ and $y_2(t)$ are independent solutions of the **homogeneous** diff.eq.

$$y''(t) + p(t)\, y'(t) + q(t)\, y(t) = 0 \quad ,$$

then a particular solution of the **inhomogeneous** diff.eq.

$$y''(t) + p(t)\, y'(t) + q(t)\, y(t) = g(t) \quad ,$$

is given by

$$Y(t) = u_1(t)y_1(t) + u_2(t)y_2(t)$$

where $u_1(t), u_2(t)$ are two functions whose derivatives satisfy the system of two equations

$$u_1'(t)y_1(t) + u_2'(t)y_2(t) = 0 \quad ,$$

$$u_1'(t)y_1'(t) + u_2'(t)y_2'(t) = g(t) \quad ,$$

Reminder about reduction of order: If $y_1(t)$ is a solution of the diff.eq. $y''(t) + p(t)y'(t) + q(t)y(t) = 0$, then to get, another, independent solution, $y_2(t)$, you write $y_2(t) = y_1(t)v(t)$, and solve the diff.eq, $y_1(t)v''(t) + (2y_1'(t) + p(t)y_1(t))v'(t) = 0$.

1. (10 pts.) Solve the initial value problem

$$y'''(t) - y''(t) + y'(t) - y(t) = 2e^t \quad , \quad y(0) = 0 \quad , \quad y'(0) = 2 \quad , \quad y''(0) = 2 \quad .$$

Ans.: $y(t) =$

Type:

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2. Verify that the given function $y_1(x)$ is a solution of the given diff.eq. , then find a second solution of the given differential equation, then write down the general solution.

$$(x-1)\,y''(x) - x\,y'(x) + y(x) = 0 \quad , \quad x > 1 \quad ; \quad y_1(x) = e^x \quad .$$

Ans.: $y(x) =$

Type:

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3. (10 pts. altogether, 5 each)
Classify the critical point $(0, 0)$ as to type, and determine whether it is is stable, asymptotically stable, or unstable, for the following systems. Explain
a. (5 pts.)

$$\mathbf{x}'(t) = \begin{pmatrix} 2 & 3 \\ 5 & 6 \end{pmatrix} \mathbf{x}(t)$$

Ans.: type: ; stability status:

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b. (5 pts.)

$$\mathbf{x}'(t) = \begin{pmatrix} 2 & 1 \\ 0 & 2 \end{pmatrix} \mathbf{x}(t)$$

Ans.: type: ; stability status:

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4. (10 pts.) Use any (correct) method to solve the initial value system

$$x_1'(t) = x_1(t) + x_2(t) \quad , \quad x_2'(t) = -x_1(t) + 3x_2(t) \quad ; \quad x_1(0) = 1 \quad , \quad x_2(0) = 1 \quad .$$

Ans.: $x_1(t) =$; $x_2(t) =$.

Type(s):

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5. (10 pts.) Set-up but do not compute a template for a particular solution of the diff.eq.

$$y'''(t) - 3y''(t) + 3y'(t) - y(t) = e^t + e^{2t} + \cos t \quad .$$

(Hint: $(r-1)^3 = r^3 - 3r^2 + 3r - 1 \quad .$)

Ans.: A template is (using A_1,A_2, etc.) $y(t) =$

Type: :

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6. (10 pts.) **Using Variation of Parameters** (no credit for other methods!), first find a particular solution of

$$y''(t) - 5y'(t) + 4y(t) = 3e^{4t} \quad ,$$

then use it to find the general solution, and finally, use the latter to solve the initial value problem

$$y''(t) - 5y'(t) + 4y(t) = 3e^{4t} \quad ; \quad y(0) = 0 \quad , \quad y'(0) = 1.$$

Ans.: $y(t) =$

Type:

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7. (10 pts.) Solve the initial value problem

$$y'''(t) - 10y''(t) + 25y'(t) = 25 \quad , \quad y(0) = 0 \quad , \quad y'(0) = 1 \quad , \quad y''(0) = 0 \quad .$$

Ans.: $y(t) =$

Type:

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8. (10 pts.) Decide whether the following functions are linearly independent or linearly dependent. In the latter case find a linear relation among them.

$$y_1(t) = 2 + \cos t \quad , \quad y_2(t) = 3 + 2\cos t \quad , \quad y_3(t) = -1 + 3\cos t \quad .$$

Ans.:

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9. (10 pts.) Find the maximal open intervals for which there exists a unique solution for initial value problems for any number in that interval

$$(t^3 - 5t^2 + 4t)\, y'''(t) + (\cos t)\, y''(t) + (t^2 + 1)\, y(t) = e^t \quad .$$

Ans.: Open Intervals:

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10. (10 pts.) Solve the initial value system

$$\mathbf{x}'(t) = \begin{pmatrix} 1 & 0 & 0 \\ 2 & 1 & -2 \\ 3 & 2 & 1 \end{pmatrix} \mathbf{x}(t) \quad , \quad \mathbf{x}(0) = \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix} \quad .$$

Ans.: $\mathbf{x}(t) =$

Type:
