Dr. Z.'s Calc4 Homework assignment 14

[Version of Nov. 10, 2016, thanks to Dafna Shochat]

1. For each of the following diff.eq. initial value problems, and intervals, decide whether the theorem promises you that there is a unique solution.

a. $(t^3+1)\,y'''(t)+(\cos t)\,y''(t)+(t^2+1)\,y(t)=\sin t \quad , \quad y(0)=1 \quad , \quad y'(0)=3 \qquad y''(0)=5 \quad ; \quad -3<t<1$

b. $(t^3+1)\,y'''(t)+(\cos t)\,y''(t)+(t^2+1)\,y(t)=\sin t \quad , \quad y(0)=1 \quad , \quad y'(0)=3 \qquad y''(0)=5 \quad ; \quad -\frac{1}{3}<t<3$

c. $y'''(t)+(\tan t)\,y''(t)+\sin t\,y(t)=\sin^2 t \quad , \quad y(0.0001)=1 \quad , \quad y'(0.0001)=3 \qquad y''(0.0001)=5 \quad ; \quad 0<t<\pi/4$

d. $y'''(t)+(\tan t)\,y''(t)+\sin t\,y(t)=\sin^2 t \quad , \quad y(0.000000001)=1 \quad , \quad y'((0.000000001)=3 \qquad y''(0.000000001)=5 \quad ; \quad 0<t<2\pi/3$

2. Find the largest open interval for which there is guaranteed to be a unique solution to

a. $(t-1)(t-2)(t+3)(t+5)\,y^{(4)}(t)+(\sin t)\,y''(t)+(\sin^2 t)\,y'(t)+(\cos t)\,y(t)=t^3 \quad ,$
$y(0)=0, y'(0)=1, y''(0)=-1, y'''(0)=4,$

b. $(t^2+1)\,y^{(4)}(t)+(\sin t)\,y''(t)+(\sin^2 t)\,y'(t)+(\cos t)\,y(t)=t^3 \quad ,$
$y(0)=0, y'(0)=1, y''(0)=-1, y'''(0)=4,$

c. $(t^2-4)\,y^{(4)}(t)+(\sin t)\,y''(t)+(\sin^2 t)\,y'(t)+\cos t\,y(t)=t^3 \quad ,$
$y(0)=0, y'(0)=1, y''(0)=-1, y'''(0)=4,$

3. Decide whether the following functions are linearly independent or linearly dependent. In the latter case find a linear relation among them.

a. $y_1(t)=1+e^t$, $y_2(t)=2+e^t$, $y_3(t)=1-e^t$

b. $y_1(t)=1+t^3$, $y_2(t)=1+t^4$, $y_3(t)=2+t^3+t^4$

c. $y_1(t)=1+t^3$, $y_2(t)=1+t^4$, $y_3(t)=3+t^3+t^4$

4.: Verify that the given functions are solutions of the diff.eq. and determine their Wronskian.

Note: (Added Oct. 26, 2014: thanks to Neha Bhat) When you 'verify' something, it may turn out to be false. If it is just say so. In these problem, if it so happens that not all proposed functions are solutions of the given diff.eq., you must say so. In that case, the second part, computing the Wronskian, is a futile excercise, that nevertheless makes sense, only it is irrelevant to the given diff. eq. .

a.

$$y'''(x) - 3y''(x) + 2y'(x) = 0 \quad ; \quad 1 \quad , \quad e^x \quad , \quad e^{2x}$$

b.

$$y'''(x) - y''(x) + y'(x) - y(x) = 0 \quad ; \quad 1 \quad , \quad \sin x \quad , \quad \cos x$$

c.

$$y'''(t) - 6y''(t) + 11y'(t) - 6y(t) = 0 \quad ; \quad e^t \quad , \quad e^{2t} \quad , \quad e^{3t}$$

d.

$$x^3y'''(x) + x^2y''(x) - 2xy'(x) + 2y(x) = 0 \quad ; \quad x \quad , \quad \frac{1}{x} \quad , \quad x^2$$