NAME: (print!) ______________________________________ **Section** ________

E-Mail address: ____

MATH 244 (1-3), Dr. Z. , FINAL EXAM, Friday, Dec. 20, 2013, 4:00-7:00pm, PH-115

No Calculators! No Cheatsheets! Write the final answer to each problem in the space provided.

Do not write below this line (office use only)

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1. (out of 20)

2. (out of 20)

3. (out of 20)

4. (out of 20)

5. (out of 20)

6. (out of 10)

7. (out of 10)

8. (out of 10)

9. (out of 10)

10. (out of 10)

11. (out of 10)

12. (out of 10)

13. (out of 10)

14. (out of 10)

15. (out of 10)

16. (out of 10)

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total: (out of 200)

You must check every answer (when applicable), and show your checking!

1. (20 pts., 10 each each)
(i) Seek a power series solution of the form $y(x) = \sum_{n=0}^{\infty} a_n x^n$ to the differential equation

$$y''(x) + y'(x) - 2y(x) = 0 \quad ,$$

and set-up a recurrence for the coefficients a_n.

Ans.: Recurrence is:

(ii) Using the recurrence that you found, find the first **three** terms of the fundamental solutions $y_1(x)$ and $y_2(x)$.

Ans.: $y_1(x) =$; $y_2(x) =$

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2. (20 pts.) Use the **Improved Euler method** to find an approximate value for $y(1.2)$ if $y(x)$ is the solution of the initial value problem ode

$$y' = x + 2y \quad , \quad y(1) = 1 \quad ,$$

using mesh-size $h = 0.1$.

Reminder:

To solve the **initial value problem**

$$y' = f(x, y) \quad , \quad y(x_0) = y_0$$

with **mesh-size** h, you define

$$x_n = x_0 + nh \quad , \quad n = 0, 1, 2, \ldots \quad ,$$

And compute, **one-step-at-a-time**

$$y_n^* = y_{n-1} + hf(x_{n-1}, y_{n-1}) \quad ,$$

$$y_n = y_{n-1} + h\frac{f(x_{n-1}, y_{n-1}) + f(x_n, y_n^*)}{2} \quad , \quad n = 1, 2 \ldots .$$

Then y_n is an **approximation** for $y(x_n)$. The smaller h, the better the approximation.

Ans.: $y(1.2)$ is approximately equal to:

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3. (20 pts.) Solve the initial value problem

$$y'''(t) - y''(t) + y'(t) - y(t) = -1 \quad ; \quad y(0) = 1 \quad , \quad y'(0) = 2 \quad , \quad y''(0) = 2 \quad .$$

Ans.: $y(t) =$

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4. (20 pts.) Use any (correct) method to solve the initial value system

$$\mathbf{x}'(t) = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \mathbf{x}(t) \quad , \quad \mathbf{x}(0) = \begin{pmatrix} 2 \\ 0 \end{pmatrix} \quad .$$

Ans.: $\mathbf{x}(t) =$

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5. (20 pts.) Set-up but do not compute a template for a particular solution of the diff.eq.

$$y^{(5)}(t) - y^{(3)}(t) = t + e^{t} + 3e^{-t} \quad .$$

Ans.: A template is (using A_1,A_2, etc.) $y(t) =$

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6. (10 pts.) Find a particular solution of the diff.eq.

$$y'''(t) - y''(t) + y'(t) - y(t) = 20e^{3t} \quad .$$

Ans.: $y(t) =$

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7. (10 pts.) Solve the initial value problem

$$y''(t) + y'(t) - 2\,y(t) = 2t - 1 \quad , \quad y(0) = 1 \quad , \quad y'(0) = -2 \quad .$$

Ans.: $y(t) =$

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8. (10 pts.) Decide whether the following functions are linearly independent or linearly dependent. Explain!

$$y_1(t) = t \quad , \quad y_2(t) = t^2 \quad , \quad y_3(t) = t^3 \quad .$$

9. (10 pts.) Find the general solution of the differential equation

$$y''(t) + y(t) = 2e^t \quad .$$

Ans.: $y(t) =$

(where c_1, c_2 are arbitrary constants)

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10. (10 pts.) Verify that the given solution $y_1(t)$ is indeed a solution of the given diff.eq. Then Find a second solution of the given differential equation. Then write down the general solution.

$$t^2\,y''(t) + 2t\,y'(t) - 2\,y(t) = 0 \quad , \quad t > 0 \quad ; \quad y_1(t) = t \quad .$$

Reminder: Suppose that you have a *general* second-order homog. linear diff.eq. (not (necessarily) with constant coefficients), that can always be written (by dividing by the coefficient of $y''(t)$, if it is not already 1):

$$y''(t) + p(t)y'(t) + q(t)y(t) = 0 \quad .$$

Suppose that someone gave you **one** solution, $y_1(t)$, then you can find another (independent!) solution, $y_2(t)$ (and hence **all** solutions, since the general solution would be $c_1y_1(t) + c_2y_2(t)$), by writing

$$y(t) = v(t)y_1(t) \quad ,$$

It is known that $v(t)$ satisfies the diff.eq.

$$y_1(t)v''(t) + (2y_1'(t) + p(t)y_1(t))v'(t) = 0 \quad .$$

Ans.: General solution is $y(t) =$

where c_1, c_2 are arbitrary constants.

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11. (10 pts.) Find the general solution of the following diff. eq.

$$y''(t) - 10\,y'(t) + 25\,y(t) = 0 \quad .$$

Ans.: $y(t) =$

(where c_1, c_2 are arbitrary constants)

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12. (10 pts.) Solve the following initial value problems. Leave the answer in **implicit form**.

$$(3x^2 + y) + (3y^2 + x)y' = 0 \quad , \quad y(1) = 1 \quad .$$

Ans.:

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13. (10 pts.) For the following diff. eq. determine the critical (equilibrium) solution(s) and decide whether they are asymptotically stable, unstable, or semi-stable.

$$\frac{dy}{dt} = y^2(y-1)(y+1)$$

Ans.:

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14. (10 pts.) Find an equation of the curve that passes through the point $(1, 2)$ and whose slope at (x, y) is x^2/y^5.

Ans.: The curve is (in **implicit form**):

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15. (10 pts.) Using the method of **integrating factor** (no credit for other methods), solve the initial value problem

$$y'(t) - 2y(t) = e^{2t} \quad , \quad y(0) = 1 \quad .$$

Ans.: $y(t) =$

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16. (10 pts.) Find out whether or not the function $y(t) = t^4 + 1$ is a solution of the initial value differential equation

$$y^{(4)}(t) = 24 \quad , \quad y(1) = 2 \quad , \quad y'(1) = 4 \quad , \quad y''(1) = 12 \quad , \quad y'''(1) = 25 \quad .$$