Solutions to the "QUIZ" for Lecture 9

1. Find $\frac{\partial f}{\partial r}$ and $\frac{\partial f}{\partial s}$ as **functions of** r **and** s , if

$$f(x, y) = x^2 + 2xy^2 + 2y^3 \quad ,$$

and the variables are related by $x = r + 2s$ and $y = 3r + 2s$. You do not need to simplify!

Solution:

$$f_r = (f_x)(x_r) + (f_y)(y_r) = (2x + 2y^2)(1) + (4xy + 6y^2)(3) = 2x + 2y^2 + 12xy + 18y^2$$

$$f_s = (f_x)(x_s) + (f_y)(y_s) = (2x + 2y^2)(2) + (4xy + 6y^2)(2) = 4x + 4y^2 + 8xy + 12y^2$$

So far this is correct, but you were asked to express everthing in terms of r and s. Plugging-in $r + 2s$ for x and $3r + 2s$ for y, we get

$$f_r = 2(r + 2s) + 2(3r + 2s)^2 + 12(r + 2s)(3r + 2s) + 18(3r + 2s)^2 \quad .$$

$$f_s = 4(r + 2s) + 4(3r + 2s)^2 + 8(r + 2s)(3r + 2s) + 12(3r + 2s)^2 \quad .$$

That's the final answers.

2. Find $\frac{\partial z}{\partial x}$ and $\frac{\partial z}{\partial y}$ if

$$x^2 + y^2 + z^2 = 5xyz + 1 \quad .$$

Solution: First move everything to the left:

$$x^2 + y^2 + z^2 - 5xyz - 1 = 0 \quad .$$

Call the left side $F(x, y, z)$. So $F(x, y, z) = x^2 + y^2 + z^2 - 5xyz - 1$. By the short-cut formulas for implicit differentiation

$$\frac{\partial z}{\partial x} = -\frac{F_x}{F_z} \quad ,$$

$$\frac{\partial z}{\partial y} = -\frac{F_y}{F_z} \quad .$$

We have

$$F_x = 2x - 5yz \quad , \quad F_y = 2y - 5xz \quad , \quad F_z = 2z - 5xy \quad .$$

So, we get

$$\frac{\partial z}{\partial x} = -\frac{2x - 5yz}{2z - 5xy} \quad ,$$

$$\frac{\partial z}{\partial y} = -\frac{2y - 5xz}{2z - 5xy} \quad .$$

That's the **answer**.