Solutions to the "QUIZ" for Lecture 6

1. Find the limit if it exists, or show that the limit does not exist.

$$\lim_{(x,y)\to(0,0)} \frac{2x}{2x+3y} \quad .$$

Sol. We first try to *plug-it-in*, and get $0/0$, which is indeterminate.

We next try to prove that the limit does **not** exist by exploring the limits when approaching $(0,0)$ along the line $y = mx$.

$$\lim_{\substack{(x,y)\to(0,0)\\ y=mx}} \frac{2x}{2x+3y} = \lim_{x\to 0} \frac{2x}{2x+3mx} =$$

$$\lim_{x\to 0} \frac{2x}{(2+3m)x} = \lim_{x\to 0} \frac{2}{(2+3m)} = \frac{2}{(2+3m)} \quad .$$

Since this **depends** on m, the limit **does not exist**, since different paths to $(0,0)$ yield different limits, and there is no consensus.

Common mistakes: **1.** $\lim_{x\to 0} \frac{2}{(2+3m)}$. This is correct, but not the final answer. The final answer should not have lim in it!

2. Find the limit if it exists, or show that the limit does not exist.

$$\lim_{(x,y)\to(0,0)} \frac{x^5}{x^2+y^2} \quad .$$

Solution: Once again it gives $0/0$. If we do as above, we would get that the limit is 0, no matter on which $y = mx$ you are travelling to $(0,0)$. So the limit **probably** exists, and if it does it is equal to 0. But this is **not** a conclusive proof. To prove it conclusively, we convert to **polar** coordinates:

$$x = r\cos\theta \quad , \quad y = r\cos\theta \quad .$$

Recall that $x^2 + y^2 = r^2$, so

$$\lim_{(x,y)\to(0,0)} \frac{x^5}{x^2+y^2} = \lim_{r\to 0} \frac{(r\cos\theta)^5}{r^2} == \lim_{r\to 0} \frac{r^5\cos^5\theta}{r^2} =$$

$$\lim_{r\to 0} r^3\cos^5\theta = \cos^5\theta \lim_{r\to 0} r^3 = \cos^5\theta \cdot 0 = 0 \quad .$$

and (if the limit is with (x,y) it can never have x or y!).