Solutions to the "QUIZ" of Lecture 4

1. Find a parametric equation for the tangent line to the curve with the given parametric equation at the specified point

$$x = \cos t \quad , \quad y = \sin t \quad , \quad z = t^2 + 1 \quad ; \quad (1, 0, 1)$$

Solution: First we write it in **vector form**

$$\mathbf{r}(t) = \langle \cos t \, , \, \sin t \, , \, t^2 + 1 \rangle \quad .$$

Next we take the derivative with respect to t:

$$\mathbf{r}'(t) = \langle -\sin t \, , \, \cos t \, , \, 2t \rangle .$$

Next we have to decide what **time** it is when the particle is at $(1, 0, 1)$. You need $\cos t = 1$, $\sin t = 0$, $t^2 + 1 = 1$. The easiest to handle is $t^2 + 1 = 1$ that gives you $t = 0$. To be on the safe side, plug-in $t = 0$ into $\mathbf{r}(t)$ and get

$$\mathbf{r}(0) = \langle \cos 0 \, , \, \sin 0 \, , \, 0^2 + 1 \rangle = \langle 1 \, , \, 0 \, , \, 1 \rangle \quad ,$$

so it agrees!

Now that you know what time it is (i.e. what is the value of t, namely $t = 0$)

$$\mathbf{r}'(0) = \langle -\sin 0 \, , \, \cos 0 \, , \, 2 \cdot 0 \rangle = \langle 0 \, , \, 1 \, , \, 0 \rangle \quad ,$$

so the **direction** of the tangent line is $\langle 0, 1, 0 \rangle$. Of course the **point** is $(1, 0, 1)$ so the **equation of the tangent line** in **compact vector form** is

$$\langle 1, 0, 1 \rangle + t \langle 0, 1, 0 \rangle \quad ,$$

and in **spelled-out vector form** it is

$$\langle 1, t, 1 \rangle \quad ,$$

and finally, in **scalar form** it is:

$$x = 1 \quad , \quad y = t \quad , \quad z = 1 \quad .$$

Ans.: The parametric equation of the tangent line to the given curve at the specified point is $x = 1$,$y = t$,$z = 1$ $(-\infty < t < \infty)$.

Note: The t of the answer is not the same as the t of the question! Every curve, including straight lines, have their own t. In other words, t is what computer scientists call a *local variable*.

Comments: Only about %70 got it completely. Some people didn't plug-in $t = 0$, but left it in terms of t getting nonsense answers like $x = -t \sin t + 1, y = t \cos t, z = 1 + 3t^2$. This is utter nonsense, since this not an equation of a straight line! Some people found the wrong t, like $t = 1$ instead of the correct $t = 0$. Some people forgot that $\sin 0 = 0$ and $\cos 0 = 1$. This should be in your blood!

2. Find $\mathbf{r}(t)$ if

$$\mathbf{r}'(t) = t\,\mathbf{i} + 2\,\mathbf{j} + (t+1)\,\mathbf{k}$$

and

$$\mathbf{r}(0) = \mathbf{i} + 2\,\mathbf{j} + 3\,\mathbf{k} \quad .$$

Solution: Integrating, we get

$$\mathbf{r}(t) = \int (t\,\mathbf{i} + 2\,\mathbf{j} + (t+1)\,\mathbf{k})dt = \frac{t^2}{2}\,\mathbf{i} + 2t\,\mathbf{j} + (\frac{t^2}{2} + t)\,\mathbf{k} + \mathbf{C} \quad .$$

We still need to find what the vector $\mathbf{C}$ is. Plugging-in $t = 0$ into the $\mathbf{r}(t)$ that we have just found, we get

$$\mathbf{r}(0) = 0\mathbf{i} + 0\,\mathbf{j} + 0\,\mathbf{k} + \mathbf{C} = \mathbf{C} \quad .$$

On the other hand, by the problem we are told that $\mathbf{r}(0) = \mathbf{i} + 2\,\mathbf{j} + 3\,\mathbf{k}$. Setting these equal we get

$$\mathbf{C} = \mathbf{i} + 2\,\mathbf{j} + 3\,\mathbf{k}.$$

Having found $\mathbf{C}$, we go back to the general form above and have

$$\mathbf{r}(t) = \frac{t^2}{2}\,\mathbf{i} + 2t\,\mathbf{j} + (\frac{t^2}{2} + t)\,\mathbf{k} + \mathbf{i} + 2\,\mathbf{j} + 3\,\mathbf{k}$$

$$= (\frac{t^2}{2} + 1)\,\mathbf{i} + (2t+2)\,\mathbf{j} + (\frac{t^2}{2} + t + 3)\,\mathbf{k} \quad .$$

This is the **answer**.

Note: Since the problem was phrased in the **i,j,k** notation the answer should also be given that way. The answer

$$= \langle \frac{t^2}{2} + 1\,,\; 2t + 2\,,\; \frac{t^2}{2} + t + 3 \rangle$$

is perfectly correct, and I doubt that you would lose any points if you put it that way, but it is still expected to use the same kind of notation for the answer as the notation in which the question was phrased.

Comments: About %80 of the students got it correct. Some people had trouble figuring-out the value of $\mathbf{C}$.