Solutions to the "QUIZ" for Lecture 3

1. Find an equation of the plane that passes through the points $(0,1,1)$, $(1,0,1)$, $(1,1,0)$.

Sol. Let's call $P = (0,1,1)$, $Q = (1,0,1)$, $R = (1,1,0)$.

We need **two** direction vectors that lie on that plane. For example

$$\mathbf{PQ} = \langle 1-0, 0-1, 1-1 \rangle = \langle 1, -1, 0 \rangle$$

$$\mathbf{PR} = \langle 1, 0, -1 \rangle$$

To get the **normal** we take the **cross-product** $\mathbf{PQ} \times \mathbf{PR}$.

$$\mathbf{PQ} \times \mathbf{PR} =$$

$$\begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 1 & -1 & 0 \\ 1 & 0 & -1 \end{vmatrix} =$$

$$\mathbf{i} \begin{vmatrix} -1 & 0 \\ 0 & -1 \end{vmatrix} - \mathbf{j} \begin{vmatrix} 1 & 0 \\ 1 & -1 \end{vmatrix} + \mathbf{k} \begin{vmatrix} 1 & -1 \\ 1 & 0 \end{vmatrix}$$

$$= \mathbf{i}((-1)\cdot(-1) - 0\cdot 0) - \mathbf{j}(1\cdot(-1) - 0\cdot 1) + \mathbf{k}(1\cdot 0 - (-1)\cdot 1) =$$

$$\mathbf{i} + \mathbf{j} + \mathbf{k} \quad .$$

Finally converting to the **usual** notation, we get $\mathbf{n} = \langle 1,1,1 \rangle$. The equation of a general plane is

$$a(x - x_0) + b(y - y_0) + c(z - z_0) = 0 \quad ,$$

where $\langle a, b, c \rangle = \langle 1,1,1 \rangle$ is the normal vector we just found, and (x_0, y_0, z_0) is *any* point (either P, Q or R). Picking P we get

$$1(x - 0) + 1(y - 1) + 1(z - 1) = 0 \quad ,$$

that simplies to

$$x + y + z = 2 \quad .$$

Ans.: $x + y + z = 2$.

Comments: About %70 got it pefectly. About %10 were clueless. The rest made some calculational mistakes. Quite a few people got $\mathbf{n} = \langle 1,1,1 \rangle$ correctly, but messed up with the last part of plugging-in $(x_0, y_0, z_0) = (0,1,1,)$, and got either $x + y + z = 0$, or $x + y + z = 3$ or other wrong things. Remember that you can always **check** your answer by plugging-in the three points and see that they lie on the planne. For $P = (0,1,1)$: $0 + 1 + 1 = 2$, for $Q = (1,0,1)$, $1 + 0 + 1 = 2$, for $R = (1,1,0)$: $1 + 1 + 0 = 2$, they all agree. People who got, for example $x + y + z = 3$ could have realized their mistake by doing this checking.

2. Find the intersection of the line

$$\mathbf{r}(t) = \langle 1, 1, 0 \rangle + t\langle 0, 2, 4 \rangle$$

and the plane

$$x + y + z = 14 \quad .$$

Solution: First **spell-out** $\mathbf{r}(t)$:

$$\mathbf{r}(t) = \langle 1, 1, 0 \rangle + t\langle 0, 2, 4 \rangle = \langle 1, 1 + 2t, 4t \rangle \quad ,$$

and in **scalar** form

$$x = 1 \quad , \quad y = 1 + 2t \quad , \quad z = 4t \quad .$$

Now plug these expressions for x, y, z in terms of the parameter t into the equation of the plane $x + y + z = 14$ getting

$$1 + (1 + 2t) + 4t = 14 \quad .$$

Simplifying, we get

$$2 + 6t = 14 \quad ,$$

so

$$6t = 12 \quad ,$$

that gives $t = 2$. Having found the lucky t (namely 2) you plug it in back into

$$x = 1 \quad , \quad y = 1 + 2t \quad , \quad z = 4t \quad ,$$

getting

$$x = 1 \quad , \quad y = 1 + 2 \cdot 2 = 5, \quad z = 4 \cdot 2 = 8 \quad ,$$

So the **lucky point** that belongs both to the plane and the line is the point $(1, 5, 8)$.

Ans.: The intersection of the line and the plane given by the problem is the point $(1, 5, 8)$.

Comments:

Some people got confused with the other type of problems, where you had to find a plane passing through three points, or the direction number of the intersection of two planes.

Do not confuse problems! It is better to admit that you don't know how to do it, then just use techniques applicable to other problems. In particular, in this problem the cross-product is completely irrelevant. Not only is it not needed, there is no way to use it!