Solutions to the "QUIZ" for Lecture 18

1. Let C be the line segment from $(0, 1)$ to $(2, 3)$, find $\int_C xy \, ds$.

Sol.: We first find a *paramteric representation* for the line segment joining $(0, 1)$ and $(2, 3)$:

$$\mathbf{r}(t) = (1 - t)\langle 0, 1 \rangle + t\langle 2, 3 \rangle = \langle 0, 1 - t \rangle + \langle 2t, 3t \rangle = \langle 2t, 1 + 2t \rangle$$

Of course **always** (for line segments): $0 \leq t \leq 1$. So $x = 2t, y = 1 + 2t$.

Next we find $\mathbf{r}'(t) : \langle 2, 2 \rangle$, so $|\mathbf{r}'(t)| = \sqrt{2^2 + 2^2} = \sqrt{8}$, and so $ds = \sqrt{8} dt$. Not we plug in $x = 2t$, $y = 1 + 2t$ into the integrand, to get:

$$\int_0^1 (2t)(1 + 2t)\sqrt{8} \, dt = \sqrt{8} \int_0^1 (2t)(1 + 2t) \, dt =$$

$$\sqrt{8} \int_0^1 2t + 4t^2 \, dt = \sqrt{8}(t^2 + 4\frac{t^3}{3})\Big|_0^1 = \sqrt{8}(1^2 + 4\frac{1^3}{3}) - 0 = \sqrt{8}\frac{7}{3} = \sqrt{2}\frac{14}{3} \quad .$$

Ans.: $\sqrt{2}\frac{14}{3}$.

2. Evaluate

$$\int_C xy^2 \, dx + x^2 y \, dy \quad ,$$

where C is $x : t^2$, $y = t^3$, $0 \leq t \leq 1$.

Sol. $dx = (t^2)' \, dt = 2t \, dt$, $dy = (t^3)' \, dt = 3t^2 \, dt$. So we have

$$\int_0^1 (t^2)(t^3)^2(2t) \, dt + (t^2)^2(t^3)(3t^2) \, dt = \int_0^1 2t^9 \, dt + 3t^9 \, dt = \int_0^1 5t^9 \, dt = \frac{t^{10}}{2}\Big|_0^1 = \frac{1}{2} \quad .$$

Ans.: $\frac{1}{2}$.