Solutions to the "QUIZ" for Lecture 11

1. Use Lagrange multipliers (no credit for other methods) to find the largest value that $x+y+z$ can be, given that $xyz = 125$

Added Oct. 13, 2009: Kin Chan pointed out that "largest" should be "smallest". The answer to the original question is DNE (Does Not exist), since one take $x = zilion$, $y = zillion$ and $z = 125/zillion^2 = verysmall$ to get $x+y+z$ (a tiny bit) larger than two zillions. The solution below is for the *smallest value*.

Solution (to the corrected question). The **goal function** is $f(x,y,z) = x+y+z$, the constraint is $xyz - 125 = 0$ so $g(x,y,z) = xyz - 125$.

$$\bigtriangledown f = \langle f_x, f_y, f_z \rangle = \langle 1, 1, 1 \rangle$$

$$\bigtriangledown g = \langle g_x, g_y, g_z \rangle = \langle yz, xz, xy \rangle$$

Setting up the **Lagrange Multipliers** equation, we have

$$\bigtriangledown f = \lambda \bigtriangledown g \quad ,$$

That, for our problem reads

$$\langle 1, 1, 1 \rangle = \lambda \langle yz, xz, xy \rangle \quad ,$$

that yield the three equations

$$1 = \lambda yz \quad , \quad 1 = \lambda xz \quad , \quad 1 = \lambda xy \quad ,$$

Dividing the first by the second gives $y = x$, and dividing the first by the third gives $x = z$, so $x = y = z$. Putting it into the constraing equation $xyz = 125$ gives $x^3 = 125$, so $x = 5$, and by back substitution, $y = 5, z = 5$. So the point is $(5, 5, 5)$. Plugging into the goal function $f(x,y,z) = x+y+z$ gives:

$$f(5,5,5) = 5+5+5 = 15 \quad .$$

Ans. The minimum value is 15. (Note: the original version said, erroneously, the "maximum value is 15, thanks again to Kin Chan). The answer to the original question is DNE.

2. You don't have to prove that it is the max. value, since the problem asked you to find the max. value, and since there is only one candidate point, that must be it!

2. Use Lagrange multipliers (no credit for other methods) to find the largest value that xyz can be, given that $x+y+z = 15$

Solution. The **goal function** is $f(x,y,z) = xyz$, the constraint is $x+y+z-15 = 0$ so $g(x,y,z) = xyz - 125$.

$$\bigtriangledown f = \langle f_x, f_y, f_z \rangle = \langle yz, xz, xy \rangle$$

$$\bigtriangledown g = \langle g_x, g_y, g_z \rangle = \langle 1, 1, 1 \rangle$$

Setting up the **Lagrange Multipliers** equation, we have

$$\bigtriangledown f = \lambda \bigtriangledown g \quad ,$$

That, for our problem reads

$$\langle yz, xz, xy \rangle = \lambda \langle 1, 1, 1 \rangle$$

that yield the three equations

$$\lambda = yz \quad , \quad \lambda = xz \quad , \quad \lambda = xy \quad ,$$

Dividing the first by the second gives $1 = y/z$ so $y = z$, dividing the first by the third gives $x = z$. So $y = x, z = x$. Plugging these into the **constraint** $x + y + z = 15$ gives $3x = 15$ so $x = 5$, and we get $y = 5, z = 5$. So the candidate point is $(5, 5, 5)$. Plugging into the goal function, we get

$$f(5, 5, 5) = 5 \cdot 5 \cdot 5 = 125 \quad .$$

Ans. The maximum value is 125.