Solutions to the "QUIZ" for Lecture 10

1. Find the local maximum and minimum point(s), the local maximum and minimum values, and saddle point(s) of the function

$$f(x,y) = 12x^2 - 4x^3 + 6y^2 + 12xy \quad .$$

Sol. First compute the first partial derivatives:

$$f_x = 24x - 12x^2 + 12y \quad , \quad f_y = 12y + 12x \quad ;$$

and for future reference, the second-order (partial) derivaties:

$$f_{xx} = 24 - 24x \quad , \quad f_{xy} = 12 \quad , \quad f_{yy} = 12 \quad .$$

To find the **critical points**, we set both f_x and f_y to 0 and solve the system of two equations and two unknwons (x and y).

$$24x - 12x^2 + 12y = 0 \quad , \quad 12y + 12x = 0 \quad .$$

Since the second one is simpler, let's treat it first, getting $y = -x$. Plugging-in $y = -x$ into the first equation gives:

$$24x - 12x^2 + 12(-x) = 0$$

that simplifies to

$$12x - 12x^2 = 0 \quad .$$

Factoring, we get

$$12x(1 - x) = 0 \quad ,$$

that yields **two** solutions $x = 0$ and $x = 1$ for the x-coordinate. Since $y = -x$, if $x = 0$ then $y = -0 = 0$ yielding the point $(0,0)$, and if $x = 1$, $y = -1$, yielding the point $(1,-1)$. So the critical points are $(0,0)$ and $(1,-1)$.

For each of them, we have to determine whether they are local max., local min. or saddle points.

For the candidate point $(0,0)$, plugging-in $x = 0$, $y = 0$ into f_{xx}, f_{xy}, f_{yy} yields:

$$f_{xx} = 24 \quad , \quad f_{xy} = 12 \quad , \quad f_{yy} = 12 \quad .$$

The **discriminant** $D = f_{xx}f_{yy} - f_{xy}^2$ equals, in this case, $24 \cdot 12 - 12^2 = 144$, and since $f_{xx} > 0$ it follows that $(0,0)$ is a **local min** (remember that if $f_{xx} > 0$ then it is a local min, while of $f_{xx} < 0$ then it is a local max.) The **local min. value** is $f(0,0) = 0$.

Now let's investigate the other candidate point $(1,-1)$. For that point

$$f_{xx} = 0 \quad , \quad f_{xy} = 12 \quad , \quad f_{yy} = 12 \quad ,$$

and $D = 0 \cdot 12 - 12^2 = -144$. Since $D < 0$, it follows that $(1, -1)$ is a **saddle point**.

Ans.: Local max: None; Local min value is 0 located at the local min point $(0, 0)$; Saddle point: $(1, -1)$.

Comments: Unfortunately, only about %30 of the students got it completely correct, but many students followed the method correctly, but sooner or later messed up the algebra. Some people "almost" got it perfectly, but said that $(0, 0)$ is a local max rather than the correct answer that it is a local min. Remember that if f_{xx} is **positive** then it is a local **min** and vice versa.

Some people need to brush up on their algebra skills. Some people solved $12x(1 - x) = 0$ and only got the solution $x = 1$. Make sure you find **all** solutions.