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1. $l.(t,0,0)$: x-axis.

2. 272.

3. $(-3,3)$

4. $-2x-12y+z+17=0$.

5. $f_{xyy}(x,y) = -4x(x^2+y)^{-3}$

6. $L(x,y)=\frac{3}{5}(x-3)+\frac{4}{5}(y-4)+5$ $L(3.001, 3.999)=4.9998$

7. doesn't exist.

8. $(-1,-1,0)$.

9. i) 6 ii) $\frac{1}{\sqrt{4e^{2t}+32\cos^2 t}}\langle 2e^t, -4\cos t, 4\cos t\rangle$

10. $\frac{\partial x}{\partial t}=10$.

Signed: Liuyang Shan.

1. $l:(t,0,0)$. x-axis

2. $h(q,r) = q^{10}\cdot r^5\cdot q^6\cdot r^{12}$

$= q^{16}\cdot r^{17}$.

$\frac{\partial h}{\partial r} = 17q^{16}r^{16}$

$\frac{\partial^2 h}{\partial r^2} = 272q^{16}r^{15}$ $\frac{\partial^2 h}{\partial r^2}(1,1) = 272$.

3. $\text{grad } f(x,y) = \langle 2x, 2y\rangle$

directional derivative of $f(x,y)$: $\langle 2x, 2y\rangle\cdot\langle 1,1\rangle = 2x+2y = 0$.

$x+y=0$.

$$\begin{cases} y = x+6 \\ y = -x \end{cases} \Rightarrow \begin{cases} x = -3 \\ y = 3 \end{cases}$$ the point is $(-3,3)$.

4. plane: $z = f(x,y)$

$f_x(x,y) = 2x$. $\vec{a} = (1, 0, 2x)$. $\vec{a}(1,2) = (1, 0, 2)$.

$f_y(x,y) = 3y^2$ $\vec{b} = (0, 1, 3y^2)$ $\vec{b}(1,2) = (0, 1, 12)$.

the normal vector is $\vec{c} = \vec{a} \times \vec{b} = -2i - 12j + k$.

so the tangent plane is $-2(x-1) - 12(y-2) + (z-9) = 0$.

$-2x - 12y + z + 17 = 0$.

5. $f_x(x,y) = \frac{2x}{x^2+y}$ $f_{xy}(x,y) = -2x(x^2+y)^{-2}$.

$f_{xyy} = -4x(x^2+y)^{-3}$

6. $f(x,y) = \sqrt{x^2+y^2}$

$f_x(x,y) = x(x^2+y^2)^{-\frac{1}{2}}$ $f_y(x,y) = y(x^2+y^2)^{-\frac{1}{2}}$

$f_x(3,4) = \frac{3}{5}$ $f_y(3,4) = \frac{4}{5}$

Linearization: $L(x,y) = \frac{3}{5}(x-3) + \frac{4}{5}(y-4) + 5$

$L(3.001, 3.999) = \frac{3}{5} \times 10^{-3} + \frac{4}{5}(-10^{-3}) + 5 = 4.9998$

7. If we close the point (1.1.1) by the line parallaling to x-axis. and getting through (1.1.1).

$$\lim_{(x,y,z)\to(1,1,1)} \frac{x+2y+3z-6}{3x+2y+z-6} = \frac{x+5-6}{3x+3-6} = \frac{x-6}{3x-3} = \frac{1}{3}$$

If we close the point (1.1.1) by the line parralleling to y-axis. and getting through (1.1.1).

$$\lim_{(x,y,z)\to(1,1,1)} \frac{x+2y+3z-6}{3x+2y+z-6} = \frac{2y+4-6}{2y+4-6} = \frac{2y-2}{2y-2} = 1$$

They are not same. so $\lim\limits_{(x,y,z)\to(1,1,1)} \frac{x+2y+3z-6}{3x+2y+z-6}$ does not exist.

8. A(1.1.0) B(-1.1.0) C(1.-1.0) D(-1.-1.0) E(0.0.1)

Put all of points into $f(x,y,z)=x+2y-z$

$f(1,1,0)=3$ $f(-1,1,0)=1$ $f(1,-1,0)=-1$ $f(-1,-1,0)=-3$ $f(0,0,1)=-1$

So. the point is (-1,-1,0)

9. i) $v(t) = \frac{dr(t)}{dt} = \langle e^t, -2\sin t, -2\sin t \rangle$

$a(t) = \frac{dv(t)}{dt} = \langle e^t, -2\cos t, -2\cos t \rangle$

$a(0) = \langle 1, -2, -2 \rangle$

$|a(0)| = 3$ $\qquad |F| = m \cdot |a| = 6\,N$

ii) $\vec{F} = m \cdot \vec{a} = \langle 2e^t, -2\cos t, -2\cos t \rangle$

$\vec{u} = \frac{1}{\sqrt{4e^{2t} + 32\cos^2 t}} \langle 2e^t, -2\cos t, -2\cos t \rangle$

10.

$$\frac{\partial f(x,y,z)}{\partial t} = \frac{\partial f(x,y,z)}{\partial x} \cdot \frac{\partial x}{\partial t} + \frac{\partial f(x,y,z)}{\partial y} \cdot \frac{\partial y}{\partial t} + \frac{\partial f(x,y,z)}{\partial z} \cdot \frac{\partial z}{\partial t}$$

$$= \frac{\partial x}{\partial t} + 2 \times 2 \times \frac{\partial x}{\partial t} + 3 \times 3 \times \frac{\partial x}{\partial t}$$

$$= 14 \times \frac{\partial x}{\partial t} =$$

$$14 \frac{\partial x}{\partial t} = 140$$

$$\frac{\partial x}{\partial t} = 10$$