NAME: (print!) _____

Section: ____ **E-Mail address:** _____

MATH 251 (22,23,24), Dr. Z. Second Chance Club for Exam 2 Worksheet

Due Friday, Dec. 13, 2020, 8:00pm. Email an attachment called scc2FirstLast.pdf to DrZcalc3@gmail.com Subject: scc2

Make sure that you have the posted solutions:

`http://www.math.rutgers.edu/~zeilberg/calc3NNN/mt2S.pdf`

and understand **each** problem.

In this worksheet, you are supposed to state what was your error for each of the exam questions (if you made an error) and say how to avoid it in the future. Then, regardless of whether you got it right or wrong do the similar problems.

Exam Question 1.

Find the Jacobian of the transformation from (u, v, w)-space to (x, y, z)-space.

$$x = uv + w \quad , \quad y = uw + v \quad , \quad z = vw + u \quad ,$$

at the point $(u, v, w) = (2, 2, 2)$.

Write down whether you got it right, and if you did not, state your error(s), and make a plan to avoid it in the future.

Regardless of whether you got them right do the following problems.

1a

Find the Jacobian of the transformation from (u, v)-space to (x, y)-space.

$$x = u^2 + v \quad , \quad y = 2u + 3uv \quad .$$

at the point $(u, v) = (1, -1)$.

1b

Find the Jacobian of the transformation from (u, v, w)-space to (x, y, z)-space.

$$x = u + 2v + 3w \quad , \quad y = -u + 2v + w^2 \quad , \quad z = v^2 + uvw \quad ,$$

at the point $(u, v, w) = (1, -1, 1)$.

Exam Question 2.

(i) (3 points) Show that

$$\mathbf{F} \;=\; \langle\, 3\,x^2yz{+}yz{+}\cos\,(x+y+z)\;,\; x^3z{+}xz{+}\cos\,(x+y+z)\;,\; x^3y{+}xy{+}\cos\,(x+y+z)\,\rangle \quad ,$$

is a conservative vector field.

(ii) (4 point) Find a function $f(x, y, z)$ such that $\mathbf{F} = \bigtriangledown f$.

(iii) (3 points) Find the line-integral $\int_C \mathbf{F}.d\mathbf{r}$ where C is the curve

$$\mathbf{r} = \langle \sin t, \cos t + 1, \sin 2t \rangle \quad , \quad 0 \leq t \leq \pi \quad .$$

Write down whether you got it right, and if you did not, state your error(s), and make a plan to avoid it in the future.

Regardless of whether you got them right do the following problems.

2a.

(i) Show that

$$\mathbf{F} \;=\; \langle\, yz\cos(xyz)\,,\; xz\cos(xyz)\,,\; xy\cos(xyz)\,\rangle \quad ,$$

is a conservative vector field.

(ii) Find a function $f(x, y, z)$ such that $\mathbf{F} = \bigtriangledown f$.

(iii) Find the line-integral $\int_C \mathbf{F}.d\mathbf{r}$ where C is the curve

$$\mathbf{r} = \langle \sin t, 3\cos t, \sin 4t \rangle \quad , \quad 0 \leq t \leq 2\pi \quad .$$

2b.

(i) (3 points) Decide whether the vector field

$$\mathbf{F} = \langle\, yz + 2y + 3z + 1\,,\; xz + 2x + 4z + 1\,,\; xy + 3x + 4y + 1\,\rangle \quad ,$$

is a conservative vector field.

(ii) It if is, find a function $f(x, y, z)$ such that $\mathbf{F} = \bigtriangledown f$.

(iii) Find the line-integral $\int_C \mathbf{F}.d\mathbf{r}$ where C is the line segment joining the points $(1, 0, 0)$ and $(0, 0, 1)$.

Exam Question 3. Sketch the region of integration and change the order of integration.

$$\int_1^2 \int_0^{e^x+1} F(x, y)\, dy\, dx$$

Write down whether you got it right, and if you did not, state your error(s), and make a plan to avoid it in the future.

Regardless of whether you got them right do the following problems.

3a. Sketch the region of integration and change the order of integration.

$$\int_1^2 \int_0^{x^3+1} F(x, y)\, dy\, dx$$

3b. Sketch the region of integration and change the order of integration.

$$\int_2^3 \int_0^{5x} F(x, y)\, dy\, dx$$

Exam Question 4. Use Largange multipliers (no credit for other methods) to find the smallest value that $x + y + z$ can be, given that $xyz = 1$.

Write down whether you got it right, and if you did not, state your error(s), and make a plan to avoid it in the future.

Regardless of whether you got them right do the following problems.

4a. Use Largange multipliers (no credit for other methods) to find the smallest value that xy can be, given that $2x + 3y = 5$.

4b. Use Largange multipliers (no credit for other methods) to find the smallest value that $2x + 3y + 4z$ can be, given that $xyz = 1$.

Exam Question 5. Compute the volume integral

$$\int\int\int_E 48\, x\, y\, z\, dV$$

where E is the region in 3D

$$\{(x, y, z)\,|\,0 \le x \le y \le z \le 1\} \quad .$$

Write down whether you got it right, and if you did not, state your error(s), and make a plan to avoid it in the future.

Regardless of whether you got them right do the following problems.

5a. Compute the volume integral

$$\int\int\int_E x^2 yz dV$$

where E is the region in 3D

$$\{(x, y, z)\,|\,0 \le y \le z \le x \le 3\} \quad .$$

5b. Compute the area integral

$$\int\int_A x^2 y dA$$

where A is the region in 2D

$$\{(x, y)\,|\,0 \le x \le 2y \le 2\} \quad .$$

Exam Question 6. By converting to polar coordinates, compute

$$\int_{-3}^{3}\int_{-\sqrt{9-x^2}}^{\sqrt{9-x^2}} \frac{\left(x^2+y^2\right)^2}{243\pi}dy\,dx$$

Write down whether you got it right, and if you did not, state your error(s), and make a plan to avoid it in the future.

Regardless of whether you got them right do the following problems.

6a. By converting to polar coordinates, compute

$$\int_{-1}^{0}\int_{-\sqrt{1-x^2}}^{\sqrt{1-x^2}} \left(x^2+y^2\right)^3 dy\,dx$$

6b. By converting to polar coordinates, compute

$$\int_{0}^{2}\int_{0}^{\sqrt{4-x^2}} xy\,(x^2+y^2)dy\,dx$$

Exam Question 7. Compute the line integral

$$\int_C \frac{4\sqrt{3}\,x\,y\,z}{3}\,ds \quad ,$$

where C is the line-segment joining $(0,0,0)$ and $(1,1,1)$

Write down whether you got it right, and if you did not, state your error(s), and make a plan to avoid it in the future.

Regardless of whether you got them right do the following problems.

7a. Compute the 3D line integral

$$\int_C x^2yz\,ds \quad ,$$

where C is the line-segment joining $(0,0,0)$ and $(-1,1,2)$

7b. Compute the 2D line integral

$$\int_C x^3y^2\,ds \quad ,$$

where C is the perimeter of the semi-disc $\{(x,y)|x^2+y^2 \le 4 \quad , x>0\}$

(Warning: that C has **two** portions).

Exam Question 8. Compute

$$\int_0^3 \int_{\sqrt{y/3}}^1 e^{x^3}\, dx\, dy \quad .$$

(Hint: Not even Dr. Z. can do $\int e^{x^3}\, dx$, so you must be clever, and first change the order of integration.)

Write down whether you got it right, and if you did not, state your error(s), and make a plan to avoid it in the future.

Regardless of whether you got them right do the following problems.

8a Compute

$$\int_0^2 \int_{y/2}^1 e^{x^2}\, dx\, dy \quad .$$

8b [Typo corrected, sorry!] Compute

$$\int_0^5 \int_{(y/5)^{1/4}}^1 e^{x^5}\, dx\, dy \quad .$$

Exam Question 9. Compute the volume integral

$$\int\int\int_E \frac{5\,(x^2+y^2+z^2)}{4\pi}\, dV \quad ,$$

where

$$E = \{(x,y,z)\,|\,x^2 + y^2 + z^2 \le 1\} \quad .$$

Write down whether you got it right, and if you did not, state your error(s), and make a plan to avoid it in the future.

Regardless of whether you got them right do the following problems.

9a Compute the volume integral

$$\int\int\int_E (x^2+y^2+z^2)^3\, dV \quad ,$$

where

$$E = \{(x, y, z) \,|\, x^2 \,+\, y^2 \,+\, z^2 \leq 1 \quad , x > 0, y > 0\} \quad .$$

9b Compute the volume integral

$$\int\int\int_E \sqrt{x^2 + y^2 + z^2}\, z \, dV \quad ,$$

where

$$E = \{(x, y, z) \,|\, x^2 \,+\, y^2 \,+\, z^2 \leq 1 \quad , z > 0\} \quad .$$

Exam Question 10. Find $\bigtriangledown \cdot \mathbf{F}$ if

$$\mathbf{F} \,=\, \langle \sin(xy)\,,\, \sin(yz)\,,\, \sin(xz) \rangle \quad .$$

Write down whether you got it right, and if you did not, state your error(s), and make a plan to avoid it in the future.

Regardless of whether you got them right do the following problems.

10a Find $\bigtriangledown \cdot \mathbf{F}$ and $\bigtriangledown \times \mathbf{F}$ if

$$\mathbf{F} \,=\, \langle x e^{xyz}, y e^{xyz}, z\cos(x + y + z) \rangle \quad .$$

10b Find $\bigtriangledown f$ if

$$f = e^{\sin(x+y+z)} \quad .$$

10c : which of the following problems are **Nonsense**? Do not evaluate them (even if they make sense)

(i) $\bigtriangledown \langle xyz, \sin(x + y), \sin(x + z) \rangle$

(ii) $\bigtriangledown \cdot \langle xyz, \sin(x + y), \sin(x + z) \rangle$

(iii) $\bigtriangledown \times \langle xyz, \sin(x + y), \sin(x + z) \rangle$

(iv): $\bigtriangledown \times \, xyz$

(v): $\bigtriangledown \cdot \, xyz$