P.1

12/10/2020

SCC2

1

$\int_C yz\,dx + (xz+z)\,dy + (xy+y+1)\,dz$

$\vec{r}(t) = [e^{t^3}, t^2 e^{t^4}, te^{t^2}] \quad 0 \le t \le 1$

$\partial/\partial x = yz \rightarrow yzx + \cancel{g(y)} \; \phi(y,z)$

$\partial/\partial y = xz + z \rightarrow xz + \phi(y,z)$

$\partial/\partial z = xy + y + 1 \qquad \hookrightarrow \phi(y,z) = z$

use fundamental Theorem of Line Integrals

$\hookrightarrow r(1) = [e\ e\ e] \quad r(0) = [1\ 0\ 0]$

$\hookrightarrow f(\text{end}) - f(\text{start})$

$= e^{3\cancel{x}} + e^{2\cancel{x}} + e$

Primary mistake was in finding the potential func and utilizing steps before FT of LI

2

$$\int_0^5 \int_{(y/5)^{1/3}}^1 \sin x^4 \, dx \, dy$$

D: $0 < y < 5$, $(y/5)^{1/3} < x < 1$

$\hookrightarrow$ $x = (y/5)^{1/3}$

$y = 5x^3$

$\downarrow$

D = $0 < x < 1$, $0 < y < 5x^3$

New Integral:

$$\int_0^1 \int_0^{5x^3} \sin x^4 \, dy \, dx$$

$$\int_0^{5x^3} \sin x^4 \, dy$$

$$\int_0^1 = 5x^3 \sin x^4 \, dx$$

$$= \frac{5}{4}(\cancel{1 - \cos 1}) \quad 1 - \cos 1$$

3 $r(u,v) = [u^3 v, uv, uv^3]$

$$\left.\begin{aligned} \partial r/\partial u &= [3u^2 v, v, v^3] \\ \partial r/\partial v &= [u^3, u, 3uv^2] \end{aligned}\right\} (1,1,1)$$

$$r_u(1,1) = [3,1,1]$$

$$r_v(1,1) = [1,1,3]$$

$$= [2,-8,2]$$

$$(x-1) - 4(y-1) + (z-1) = 0$$

$$z = -x + 4y - 2$$

4

$$f(x,y,z) = e^{\cos x^2} + \sin xyz + \cos xz$$

$$r(t) = [\cos t, t, \sin t]$$

$\vec{F} = \text{grad}(f) \rightarrow$ use FToLI

$$e^{\cos 1 + 1} - e^{\cos 1 + 1} = 0$$

5

$$\int_R (x^2+y^2+z^2)^3 \, dx \, dy \, dz$$

$(x,y,z) \mid x^2+y^2+z^2 \leq 1, \; x,y,z \geq 0$

Turn to polar coords (spherical)

$\rho, \phi, \theta \Rightarrow \rho < 1, \; 0 < \phi < \pi/2, \; 0 < \theta < \pi/2$

$$\int_0^{\pi/2} \int_0^{\pi/2} \int_0^1 \rho^8 \sin\phi \, d\rho \, d\phi \, d\theta$$

$\downarrow$

B/c
$x^2+y^2+z^2=\rho^2$
$\& \; (\rho^2)^3 \rho^2$

$$\left(\int_0^1 \rho^8 \, d\rho\right)\left(\int_0^{\pi/2} 1 \, d\theta\right)\left(\int_0^{\pi/2} \sin\phi \, d\phi\right)$$

$$= \frac{1}{9} \cdot \frac{\pi}{2} = \frac{\pi}{18}$$

6 $$\int_{-3}^{0}\int_{0}^{\sqrt{9-x^2}} (x^2+y^2)^2 \, dy \, dx$$

Again... use polar coords

$(x,y) \to -3 < x < 0, \; 0 < y < \sqrt{9-x^2}$

$(r,\theta) = (0 < r < 3, \; \frac{\pi}{2} < \theta < \pi)$

$(r^2)^2 = r^4(r) = r^5$

$$\int_{0}^{3}\int_{\pi/2}^{\pi} r^5 \, d\theta \, dr = 243/4 \, \pi$$

7 lim 1 is just simple. Plug in vals to get lim of $\sqrt{\pi}$

lim 2 can be simplified to x+y in order to get the ans of 0

lim 3 & 4 DNE b/c they return in the indeterminate form in every way or are nonsensical

$(0,0,0)(1,2,-3)$

$f(x,y,z) = xyz$

$\int_C f \cdot ds$

$r(t) = [t, 2t, -3t]$

$r'(t) = [1, 2, -3]$

$|r'(t)| = \sqrt{14}\,dt = ds$

$\int_0^1 -6t^3 \sqrt{14}\, dt$

$\left[-\sqrt{14}\ \frac{3t^4}{2}\right]_0^1$

$= -\frac{3\sqrt{14}}{2}$

9 $F = [z, z, x]$

$z = 9 - x^2 - y^2, \ x \geq 0, y \geq 0, z \geq 0$

$\frac{\partial g}{\partial x} = -2x$

$\frac{\partial g}{\partial y} = -2y$

$P = Q = z,\ R = x$

$$\iint ((-z)(-2x) - z(-2y) + x)\, dA$$

$$= \int_0^3 \int_0^{\sqrt{9-x^2}} ((2x+2y)z + x)\, dy\, dx$$

$= 693/5$

$= -693/5$

10

$f(x,y,z) = xyz$

$\nabla f = \lambda \nabla g$

$$\begin{bmatrix} yz \\ xz \\ xy \end{bmatrix} = \lambda \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}$$

$$\left.\begin{aligned} yz &= \lambda \\ xz &= \lambda 2 \\ xy &= \lambda 3 \end{aligned}\right\} \begin{aligned} &\cancel{(x,y,z)} \\ &x,y,z = 2,3,6 \end{aligned}$$

Very General Comments on where I went wrong...

For Line Integrals, it would be useful to more appropriately analyze the problem & determine its TYPE & method to approach.

I made the bulk of my mistakes on Line & Surface Integrals, both of which I need to work on problem approach & methodology.