SCC II Worksheet Shaun Goda

Exam Problem 1.

Compute the line integral $\int_C yz\,dx + (xz+z)\,dy + (xy+y+1)\,dz$

over the path $r(t) = \langle e^{t^3}, t^2e^{t^4}, te^{t^7} \rangle$, $0 \le t \le 1$

$$\int_C yz\,dx = \int_0^1 (t^3e^{t^4+t^7})(3t^2e^{t^3})\,dt = \int_0^1 3t^5e^{t^7+t^4+t^3}\,dt$$

$$\approx 3.7256$$

$$\int_C (xz+z)\,dy = \int_0^1 (te^{t^7+t^3} + te^{t^7})(4t^5e^{t^4} + 2te^{t^4})\,dt$$

$$\approx 11.6777$$

$$\int_C (xy+y+1)\,dz = \int_0^1 (t^2e^{t^4+t^3} + t^2e^{t^4} + 1)(7e^{t^7}t^7 + e^{t^7})\,dt$$

$$\approx 14.7896$$

$$\int_C yz\,dx + \int_C (xz+z)\,dy + \int_C (xy+y+1)\,dz = \boxed{30.1929}$$

What did I do wrong?

I did not memorize the formula for the line integral with the format $\int P\,dx + Q\,dy + R\,dz$.

Problem 1a.

I did it the wrong way here.
This problem can be solved using fundamental theorem.

Compute the line integral $\int_C xe^{xyz}\,dx + ye^{xyz}\,dy + ze^{xyz}\,dz$

Over the path $r(t) = \langle t, t^2, t^3 \rangle$, $0 \le t \le 1$

Same for this one

$$\int_C xe^{xyz}\,dx = \int_0^1 (te^{t^6})(1)\,dt \approx 0.67095$$

$$\int_C ye^{xyz}\,dy = \int_0^1 (t^2e^{t^6})(2t) \approx 0.78120$$

$$\int_C z e^{xyz} dz = \int_0^1 (t^3 e^{t^6})(3t^2)\, dt = \frac{e-1}{2} \approx 0.85914$$

$$\int_C x e^{xyz} dx + \int_C y e^{xyz} dy + \int_C z e^{xyz} dz \approx \boxed{2.31129}$$

Problem 1b

$$\int_C \nabla f \cdot dr = \int_{a_t}^{b_t} \left(f_x \frac{dx}{dt} + f_y \frac{\partial y}{\partial t} + f_z \frac{\partial z}{\partial t}\right) dt = f(r(t_b)) - f(r(t_a))$$

Compute the line integral $\int_C (4x^3y^2+1)dx + (2x^4y+1)dy$
over the path $r(t) = \langle \sin t^2, \cos t^2 \rangle$, $0 \le t \le \sqrt{\frac{\pi}{2}}$

find f when $\nabla f = \langle \overset{4x^3y^2+1}{\cancel{\sin t}}, \overset{2x^4y+1}{\cancel{\cos t}} \rangle$,

↖ I fixed it here.

f will be $x^4y^2 + x + y$

plug in $r(t)$

$f(r(t)) = \sin^4 t^2 \cos^2 t^2 + \sin t^2 + \cos t^2$

$$\int_C (4x^3y^2+1)dx + (2x^4y+1)dy = f(r(\sqrt{\tfrac{\pi}{2}})) - f(r(0))$$

$$= \sin^4(\sqrt{\tfrac{\pi}{2}})^2 \cos^2(\sqrt{\tfrac{\pi}{2}})^2 + \sin(\sqrt{\tfrac{\pi}{2}})^2 + \cos(\sqrt{\tfrac{\pi}{2}})$$

$$- \sin^4(0)\cos^2(0) - \sin(0)^2 - \cos(0)^2$$

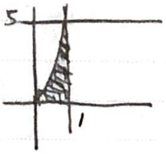
$$\cancel{\approx}\ 1 - 1 = \boxed{0}$$

Exam Problem 2.

By changing the order of integration, if necessary, evaluate the double integral

$$\int_0^5 \int_{\sqrt[3]{\frac{y}{5}}}^1 \sin x^4 \, dx\, dy$$

$$\int_0^5 \int_{\sqrt[3]{\frac{y}{5}}}^1 \sin x^4 \, dx\, dy = \int_0^1 \int_0^{5x^3} \sin x^4 \, dy\, dx$$



$x = \sqrt[3]{\frac{y}{5}}$

$x^3 = \frac{y}{5}$

$y = 5x^3$

$$\Rightarrow \int_0^{5x^3} \sin x^4 \, dy = \left| \sin x^4 y \right|_0^{5x^3} = \sin x^4 \, 5x^3$$

$$\int_0^1 \sin(x^4) 5x^3 \, dx = \left| -\frac{5\cos(x^4)}{4} \right|_0^1$$

$$\boxed{= -\frac{5}{4}\cos(1) + \frac{5}{4} \approx 0.57462}$$

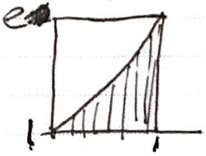
Problem 2a.

Change the order of integration of $\int_0^1 \int_0^{e^x} f(x,y)\, dy\, dx$

$$\int_0^1 \int_0^{e^x} f(x,y)\, dy\, dx = \int_0^e \int_{\ln(y)}^1 f(x,y)\, dx\, dy$$

$y = e^x$

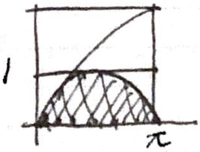
$x = \ln(y)$



Problem 2b

Change the order of integration of $\int_0^{\pi} \int_0^{\sin x} f(x,y)\, dy\, dx$

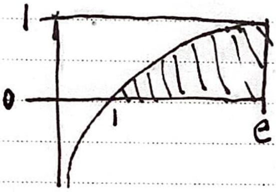
$$\int_0^{\pi} \int_0^{\sin x} f(x,y)\, dy\, dx = 2\int_0^1 \int_0^{\arcsin x} f(x,y)\, dx\, dy$$



Problem 2C

Change the order of integration of $\int_0^1 \int_{e^y}^{e} f(x,y)\,dx\,dy$

$$\int_0^1 \int_{e^y}^{e} f(x,y)\,dx\,dy = \int_1^e \int_0^{\ln(x)} f(x,y)\,dy\,dx$$



$e^y = x$

$y = \ln(x)$

Exam Problem 3

Find the equation of the tangent plane at the point (1, 1, 1) to the surface given parametrically by

$x(u,v) = u^3 v$, $y(u,x) = uv$, $z(u,v) = uv^3$

$-\infty < u < \infty$, $-\infty < v < \infty$

$r = u^3 v i + uv j + uv^3 k$

$r_u = 3u^2 v i + v j + v^3 k$

$r_v = u^3 i + u + 3uv^2 k$

$r_u(1,1) = 3i + j + k = \langle 3, 1, 1 \rangle$

$r_v(1,1) = i + j + 3k = \langle 1, 1, 3 \rangle$

$$\begin{vmatrix} i & j & k \\ 3 & 1 & 1 \\ 1 & 1 & 3 \end{vmatrix} = (3-1)i - (9-1)j + (3-1)k = \langle 2, -8, 2 \rangle$$

$\langle 2, -8, 2 \rangle \cdot \langle x-1, y-1, z-1 \rangle = 2x - 2 - 8y + 8 + 2z - 2 = 0$

$2z = 8y - 2x - 4$

$\boxed{z = 4y - x - 2}$

What I did wrong?

I was stumbled by $-\infty \le u \le \infty$. this has no effect.

Problem 3a.

$x(u,v)=u^2,\ y(u,v)=uv,\ z(u,v)=v^2$

$r=\langle u^2, uv, v^2\rangle$

$r_u=\langle 2u, v, 0\rangle$

$r_v=\langle 0, u, 2v\rangle$

$r_u(1,2)=\langle 2, 2, 0\rangle$

$r_v(1,2)=\langle 0, 1, 4\rangle$

$$\begin{vmatrix} i & j & k \\ 2 & 2 & 0 \\ 0 & 1 & 4 \end{vmatrix} = (2-0)i-(8-0)j+(2-0)k$$

$$=\langle 2, -8, 2\rangle$$

$\langle 2, -8, 2\rangle \cdot \langle x-1, y-2, z-4\rangle$

$= 2x-2-8y+16+2z-8 = 2x-8y+2z=-6$

$2z = 8y-2x-6$

$\boxed{z = 4y-x-3}$

Problem 3b.

$x(u,v)=u^3,\ y(u,v)=v^3\ z(u,v)=-2uv$ @ $(-1,-1,2)$

$r=\langle u^3, v^3, -2uv\rangle$

$r_u=\langle 3u^2, 0, -2v\rangle$

$r_v=\langle 0, 3v^2, -2u\rangle$

$r_u(-1,-1)=\langle 3, 0, 2\rangle$

$r_v(-1,-1)=\langle 0, 3, 2\rangle$

$$\begin{vmatrix} i & j & k \\ 3 & 0 & 2 \\ 0 & 3 & 2 \end{vmatrix} = (0-6)i-(6-0)j+(9-0)k$$

$$=\langle -6, -6, 9\rangle$$

$\langle -6, -6, 9\rangle \cdot \langle x+1, y+1, z-2\rangle$

$= -6x-6-6y-6+9z-18 = -6x-6y+9z-30$

$9z = 6x+6y+30$

$\boxed{z = \frac{2}{3}x+\frac{2}{3}y+\frac{10}{3}}$

Exam Problem 4.

$f(x, y, z) = e^{\cos x^2 + \sin xyz + \cos xz}$

where $F = \langle \frac{\partial f}{\partial x}, \frac{\partial f}{\partial y}, \frac{\partial f}{\partial z} \rangle$, $r(t) = \langle \cos t, t, \sin t \rangle$ $0 \le t \le 2\pi$

use fundamental theorem of line integal.

$r(2\pi) = \langle 1, 2\pi, 0 \rangle$

$r(0) = \langle 1, 0, 0 \rangle$

$e^{\cos 1 + \sin 0 + \cos 0} - e^{\cos 1 + \sin 0 + \cos 0}$

$e^{\cos(1) + 1} - e^{\cos 1 + 1} = \boxed{0}$

what did I do wrong?

I was not able to recognize to use the fundamental theorem of line integral.

Problem 4a.

$r(3) = \langle 3, 9, 27 \rangle$

$r(0) = \langle 0, 0, 0 \rangle$

$\sin(3 + 81 + 27^3) - \sin(0)$

$= \boxed{\sin(19767) \approx 0.09886}$

Problem 4b.

$r(\pi) = \langle \sin 2\pi, \cos \pi \rangle = \langle 0, -1 \rangle$

$r(0) = \langle \sin 0, \cos 0 \rangle = \langle 0, 1 \rangle$

$e^{\cos(0) + 3\sin(-1)} - e^{\cos(0) + 3\sin(1)}$

$= \boxed{e^{1 + 3\sin(-1)} - e^{1 + 3\sin(1)} \approx -33.716}$

Exam Problem 5

Evaluate the triple integral

$\int_R (x^2 + y^2 + z^2)^3 \, dx\,dy\,dz$ where

$R = \{(x, y, z) \mid x^2 + y^2 + z^2 \le 1, \ xyz \ge 0\}$

$$\int_R p^8 \sin\phi \, dp\, d\phi\, d\theta = \int_0^{\frac{\pi}{2}} \int_0^{\frac{\pi}{2}} \int_0^1 p^8 \sin\varphi \, dp\, d\varphi\, d\theta$$

$$\int_0^1 p^8 \sin\varphi \, dp = \left[\frac{p^9}{9} \sin\varphi \right|_0^1 = \frac{1}{9} \sin\varphi$$

$$\frac{1}{9} \int_0^{\frac{\pi}{2}} \sin\varphi \, d\varphi = \frac{1}{9} \left| -\cos\varphi \right|_0^{\frac{\pi}{2}} = \frac{1}{9}$$

$$\int_0^{\frac{\pi}{2}} \frac{1}{9} \, d\theta = \boxed{\frac{\pi}{18}}$$

What I did wrong?

I messed up the integral calculation !!

Problem 5a.

$$\int_R (x+y)(x^2+y^2+z^2)^2 \, dx\, dy\, dz$$

$$= \int_0^{\pi} \int_0^{\pi} \int_0^1 (p \sin\phi \cos\theta + p \sin\phi \sin\theta) \, p^4 \sin\phi \, dp\, d\phi\, d\theta$$

$$p^5 \sin^2\phi \cos\theta + p^5 \sin^2\phi \sin\theta$$

$$p^5 \sin^2\phi (\cos\theta + \sin\theta)$$

$$\int_0^{\pi} \int_0^{\pi} \int_0^1 (p^5 \sin^2\phi \cos\theta + p^5 \sin^2\phi \sin\theta) \, dp\, d\phi\, d\theta$$

$$\int_0^1 (p^5 \sin^2\phi \cos\theta + p^5 \sin^2\phi \sin\theta) \, dp = \frac{1}{6} \sin^2\phi \cos\theta + \frac{1}{6} \sin^2\phi \sin\theta$$

$$\int_0^{\pi} \left(\frac{1}{6} \sin^2\phi \cos\theta + \frac{1}{6} \sin^2\phi \sin\theta\right) d\phi = \frac{\pi}{12} (\cos\theta + \sin\theta)$$

$$\frac{\pi}{12} \int_0^{\pi} (\cos\theta + \sin\theta) \, d\theta = \boxed{\frac{\pi}{6}}$$

Problem 5b.

$$\int_R z(x^2+y^2+z^2)\,dx\,dy\,dz$$

$$= \int_{\pi}^{2\pi}\int_0^{\pi}\int_0^{1} \rho^3\cos\phi \; d\rho\, d\phi\, d\theta$$

$$= \int_0^1 \rho^3\cos\phi\, d\rho = \left|\frac{\rho^4}{4}\cos\phi\right|_0^1 = \frac{1}{4}\cos\phi$$

$$= \frac{1}{4}\int_0^{\pi}\cos\phi\, d\phi = \frac{1}{4}\left|\sin\phi\right|_0^{\pi} = \boxed{0}$$

Problem 5c.

$$\int_R (z-x)\,dx\,dy\,dz$$

$$= \int_0^{2\pi}\int_0^{\pi}\int_0^{\sqrt{8}} (\rho\cos\phi - \rho\sin\phi\cos\theta)\rho^2\sin\phi\; d\rho\, d\phi\, d\theta$$

$$= \int_0^{2\pi}\int_0^{\pi}\int_0^{\sqrt{8}} (\rho^3\cos\phi\sin\phi - \rho^3\sin^2\phi\cos\theta)\, d\rho\, d\phi\, d\theta$$

$$\int_0^{\sqrt{8}} (\rho^3\cos\phi\sin\phi - \rho^3\sin^2\phi\cos\theta)\, d\rho = 16\cos_{\phi}\sin_{\phi} - 16\sin^2_{\phi}\cos_{\theta}$$

$$16\int_0^{\pi} (\cos\phi\sin\phi - \sin^2\phi\cos\theta)\, d\phi = -8\pi\cos\theta$$

$$-8\pi\int_0^{2\pi}\cos\theta\, d\theta = \boxed{0}$$

Exam Problem 6.

Evaluate the double integral

$$\int_{-3}^{0}\int_0^{\sqrt{9-x^2}} (x^2+y^2)^2\, dy\, dx$$

$$\int_0^3\int_{\frac{\pi}{2}}^{\pi} r^5\, d\theta\, dr \Rightarrow \int_{\frac{\pi}{2}}^{\pi} r^5\, d\theta = \pi r^5 - \frac{\pi}{2}r^5$$

$$\int_0^3 \left(\pi r^5 - \frac{\pi}{2}r^5\right)dr = \frac{243}{2}\pi - \frac{243}{4}\pi = \boxed{\frac{243}{4}\pi}$$

What I did wrong?

I did get the question right!!

But I did not use polar coordinates.

Problem 6~~a.~~ b.

$$\int_0^4 \int_{-\sqrt{16-x^2}}^{0} (x^2+y)\,dy\,dx$$ find integral to polar coordinate

$$\int_0^4 \int_{\frac{3\pi}{2}}^{2\pi} (r^2\cos^2\theta + r\sin\theta)\, r\, d\theta\, dr$$

Problem 6~~b~~.a.

$$\int_{-3}^{0} \int_0^{\sqrt{9-x^2}} (x^2+y)\,dy\,dx$$ find integral to polar coordinate.

$$\int_0^3 \int_{\frac{\pi}{2}}^{\pi} (r^2\cos^2\theta + r\sin\theta)\, r\, d\theta\, dr$$

Problem 6c.

$$\int_{-\frac{\sqrt{2}}{2}}^{\frac{\sqrt{2}}{2}} \int_x^{\sqrt{1-x^2}} (x^3+y^2)\,dy\,dx$$ find the integral to polar coordinate.

$$\int_0^1 \int_{\frac{\pi}{4}}^{\frac{3\pi}{4}} (r^3\cos^3\theta + r^2\sin^2\theta)\, r\, d\theta\, dr$$

☆ Exam Problem 7.

Decide whether limit exist or not

(a) $$\lim_{(x,y)\to(\frac{\pi}{2},\frac{\pi}{2})} \frac{\cos x+\sin x}{x+y} = \frac{\cos(\frac{\pi}{2})+\sin(\frac{\pi}{2})}{\frac{\pi}{2}+\frac{\pi}{2}} = \boxed{\frac{1}{\pi}}$$

(b) $$\lim_{(x,y)\to(0,0)} \frac{x^2-y^2}{x-y} = \frac{0}{0}$$

$$\frac{x^2-y^2}{x-y} = \frac{\cancel{(x-y)}(x+y)}{\cancel{(x-y)}} = (x+y)$$

$$\lim_{(x,y)\to(0,0)} (x+y) = \boxed{0}$$

(c) $\lim_{(x,y)\to(0,0)} \frac{x-y}{x^2-y^2} = \frac{0}{0}$

$\frac{x-y}{x^2-y^2} = \frac{\cancel{(x-y)}}{(x+y)\cancel{(x-y)}} = \frac{1}{x+y}$

$\lim_{(x,y)\to(0,0)} \frac{1}{x+y} = \frac{1}{0}$ $\boxed{\text{DNE}}$

(d) $\lim_{(x,y)\to(1,1)} \frac{x+y-2}{2x+y-3} = \frac{0}{0}$

$\lim_{(x,y)\to(1+t,1)} \frac{x+y-2}{2x+y-3} = \frac{1+t+1-2}{2+2t+1-3} = \frac{t-2}{2t}$

$= \frac{1}{2} - \frac{1}{t}$ $\lim_{t\to0}\left(\frac{1}{2}-\frac{1}{t}\right) = \frac{1}{2}$

$(x,y)\to(1,1+t) = \frac{1+1+t-2}{2+1+t-3} = \frac{t}{t} = 0$

$\frac{1}{2} \neq 0$ $\boxed{\text{DNE}}$

What I did wrong? did not show work for DNE

Problem 7a.

$\lim_{(x,y)\to(1,3)} \frac{x-1}{\cancel{y}-3} = \frac{0}{0}$

$(x,y)\to(1+t,1) = \frac{1+t-1}{1-3}$ $\frac{t}{-2}$ $\lim_{t\to0} = 0$

$(x,y)\to(1,1+t) = \frac{1-1}{} = 0$ $0 = 0$

$\lim_{(x,y)\to(0.999,2.999)} = 1$

$\lim_{(x,y)\to(1.001,3.001)} = 1$

$\lim_{(x,y)\to(1,3)^-} = \lim_{(x,y)\to(1,3)^+}$ $\boxed{\text{answer is } 1}$

Problem 7b.

$$\lim_{(x,y,z)\to(0,0,0)} \frac{x+y+2z}{2x+y+z} = \frac{0}{0}$$

$$\lim_{(x,y,z)\to(-0.001,-0.001,-0.001)} \frac{x+y+2z}{2x+y+z} = 1$$

$$\lim_{(x,y,z)\to(0.001,0.001,0.001)} \frac{x+y+2z}{2x+y+z} = 1$$

$\lim^{+} = \lim^{-}$ $\boxed{\text{answer is } 1}$

Exam Problem 8

Compute the line integral $\int_C f\,ds$ where $f(x,y,z) = xyz$ and the line segment is from $(0,0,0)$ to $(1,2,-3)$

$\vec{r}(t) = (1-t)\langle 0,0,0\rangle + t\langle 1,2,-3\rangle$

$r(t) = \langle t, 2t, -3t\rangle$ for $0 \le t \le 1$

$$\int_C f\,ds = \int_0^1 (t)(2t)(-3t)\sqrt{1^2+2^2+3^2}\,dt$$

$$= \int_0^1 -6t^3(\sqrt{14})\,dt = -\frac{[illegible]}{6\sqrt{14}} \int \frac{t^4}{4}\Big|_0^1 = \boxed{[illegible]\ -\frac{3}{2}\sqrt{14}}$$

Problem 8a.

find $\int_C f\,ds$ for $f(x,y,z) = xy^2 + yz^2 + z$ where C is from $(0,0,0)$ to $(1,1,-1)$

$r(t) = (1-t)\langle 0,0,0\rangle + t\langle 1,1,-1\rangle$

$r(t) = \langle t, t, -t\rangle$ for $0 \le t \le 1$

$$\int_C f\,ds = \int_0^1 \underset{t^3+t^3-t}{([illegible])}\sqrt{1^2+1^2+1^2}\,dt$$

$$= \sqrt{3}\int_0^1 (2t^3 - t)\,dt = \sqrt{3}\left|\frac{t^4}{2} - \frac{t^2}{2}\right|_0^1$$

$$= \sqrt{3}\left(\frac{1}{2} - \frac{1}{2}\right) = \boxed{0}$$

☆ Problem 8b.

find $\int_C f\,ds$ where $f(x,y) = x+y$

for $\{(x,y) : x^2+y^2=1 \quad y>0\}$

$r(t) = (1-t)\langle 1,0\rangle + t\langle -1,0\rangle$

$= \langle 1-t, 0\rangle + \langle -t, 0\rangle$

$= \langle 1-2t, 0\rangle$ for $0 \le t \le 1$

$$\int_C f\,ds = 2\int_0^1 (1-2t)\,dt$$

$$= \int_0^1 (2-4t)\,dt = \Big| 2t - 2t^2 \Big|_0^1 = 2-2 = 0$$

$\boxed{\text{answer is } 0}$

Exam Problem 9.

Compute the vector-field surface integral $\iint_S F\cdot ds$ if F is $\langle z, z, x\rangle$ where S is the oriented surface $z = 9-x^2-y^2$, $x\ge 0, y\ge 0, z\ge 0$ with downward pointing normal.

$g(x,y) = 9-x^2-y^2$

$g_x = -2x$

$g_y = -2y$

$$\iint_S F\cdot ds = \iint_D (2xz+2yz+x)\,dA$$

$$= \iint_D ((2x+2y)(9-x^2-y^2)+x)\,dA$$

$$= \int_0^3 \int_0^{\sqrt{9-x^2}} ((2x+2y)(9-x^2-y^2)+x)\,dy\,dx$$

use eval f answer $= \frac{693}{3} * -1 = \boxed{\frac{693}{3}}$

What I did wrong?

I ran out of time!! I panicked!

Problem 9a.

find $\iint_S F \cdot ds$ when $F = \langle x+z, y+z, -x \rangle$
and S is $z = 9 - x^2 - y^2$, $x<0$, $y<0$, $z \geq 0$

$g(x,y) = 9 - x^2 - y^2$
$g_x = -2x$
$g_y = -2y$

$$\iint_S F \cdot ds = \iint_D (-(x+z)(-2x) - (y+z)(-2y) - x)\, dA$$

$$= \int_{-3}^{0} \int_{\sqrt{9-x^2}}^{0} (-(x+9-x^2-y^2)(-2x) - (y+9-x^2-y^2)(-2y) - x)\, dy\, dx$$

use evalf answer $= \boxed{\left(-\frac{81\pi}{4} - 9\right)}$

Problem 9b

find $\iint_S F \cdot ds$ when $F = \langle x+z, y+x, -x \rangle$
and S is $z = 9 - x^2 - y^2$, $0<x<1$, $0<y<1$, $z \geq 0$

$$\iint_S F \cdot ds$$

$$= \int_0^1 \int_0^1 (-(x+9-x^2-y^2)(-2x) - (y+9-x^2-y^2)(-2y) - x)\, dy\, dx$$

$= \frac{103}{6}$ answer $= \frac{103}{6}(-1) = \boxed{-\frac{103}{6}}$

Exam Problem 10

Find the point on the plane $x + 2y + 3z = 18$ where the function $f(x,y,z) = xyz$ is as large as possible.

$f(x,y,z) = xyz$ $\nabla f = \langle yz, xz, xy \rangle$
$g(x,y,z) = x + 2y + 3z - 18$ $\nabla g = \langle 1, 2, 3 \rangle$

$\nabla f = \lambda \nabla g$

$\langle yx, xz, zy \rangle = \lambda \langle 1, 2, 3 \rangle$

$yx = \lambda \quad \times \quad xz = 2\lambda \quad \times \quad zy = 3\lambda$

$x^2y^2z^2 = 6\lambda^3$

$x^2(\lambda^2 = 6\lambda \qquad y^2\lambda^2 4 = 6\lambda^3 \qquad z^2\lambda^2 9 = 6\lambda^3$

$x^2 = 6\lambda \qquad y^2 = \frac{3}{2}\lambda \qquad z^2 = \frac{2}{3}\lambda$

$x = \sqrt{6\lambda} \qquad y = \sqrt{\frac{3}{2}\lambda} \qquad z = \sqrt{\frac{2}{3}\lambda}$

$\sqrt{6\lambda} + 2\sqrt{\frac{3}{2}\lambda} + 3\sqrt{\frac{2}{3}\lambda} = 18$

$\sqrt{\lambda} = \sqrt{6} \qquad \lambda = 6$

answer $= (\sqrt{6 \cdot 6}, \sqrt{\frac{3}{2}6}, \sqrt{\frac{2}{3}6}) = \boxed{(6, 3, 2)}$

Problem 10a.

find the maximum value of the function $f(x,y,z) = xyz$ on the plane $2x+y+z=4$

$f(x,y,z) = xyz \qquad \nabla f = \langle yz, xz, xy \rangle$

$g(x,y,z) = 2x+y+z-4 \qquad \nabla g = \langle 2, 1, 1 \rangle$

$\nabla f = \lambda \nabla g$

$\langle yz, xz, xy \rangle = \lambda \langle 2, 1, 1 \rangle$

$yz = 2\lambda \quad \times \quad xz = \lambda \qquad xy = \lambda$

$x^2y^2z^2 = 2\lambda^3$

$x^2(4\lambda^2) = 2\lambda^3 \qquad y^2(\lambda^2) = 2\lambda^3 \qquad z^2(\lambda^2) = 2\lambda^3$

$x^2 = \frac{1}{2}\lambda \qquad y^2 = 2\lambda \qquad z^2 = 2\lambda$

$x = \sqrt{\frac{1}{2}\lambda} \qquad y = \sqrt{2\lambda} \qquad z = \sqrt{2\lambda}$

$2\sqrt{\frac{1}{2}\lambda} + \sqrt{2\lambda} + \sqrt{2\lambda} = 4$

$3\sqrt{2\lambda} = 4 \Rightarrow \sqrt{2\lambda} = \frac{4}{3} \Rightarrow 2\lambda = \frac{16}{9} \Rightarrow \lambda = \frac{8}{9}$

answer $= (\sqrt{\frac{1}{2} \cdot \frac{8}{9}}, \sqrt{2 \cdot \frac{8}{9}}, \sqrt{2 \cdot \frac{8}{9}})$

$= (\sqrt{\frac{4}{9}}, \sqrt{\frac{16}{9}}, \sqrt{\frac{16}{9}})$

$= \boxed{(\frac{2}{3}, \frac{4}{3}, \frac{4}{3})}$

Problem 10b

Find the point on the plane $2x+y+z=4$ where $f(x,y,z)=xy^2z$ is as large as possible

$f(x,y,z) = xy^2z$ $\quad \nabla f = \langle y^2z,\ 2xyz,\ xy^2 \rangle$

$g(x,y,z) = 2x+y+z-4$ $\quad \nabla g \langle 2, 1, 1 \rangle$

$\nabla f = \lambda \nabla g$

$\langle y^2z,\ 2xyz,\ xy^2 \rangle = \lambda \langle 2, 1, 1 \rangle$

$2\lambda = y^2z$ $\quad \lambda = 2xyz$ $\quad \lambda = xy^2$

$2x^2y^5z^2 = 2\lambda^3$

$x^2y^5z^2 = \lambda^3$

$y^2 = \frac{2\lambda}{z}$

$\frac{\lambda^2y^3}{4} = \lambda^3$

$y^3 = \cancel{\ } 4\lambda$

$y = \sqrt[3]{4\lambda}$

$x^2y(4\lambda^2) = \lambda^3$

$x^2y = \frac{\lambda}{4}$

$4x^2y = \lambda$

$\lambda^2yz^2 = \lambda^3$

$yz^2 = \lambda$

$\lambda = x(4\lambda)^{\frac{2}{3}}$

$x = \frac{\lambda}{(4\lambda)^{\frac{2}{3}}}$

$4x^2y = yz^2$

$4x^2 = z^2$

$2x = z$

$4x^2y = \lambda$

$x^2y = \frac{\lambda}{4}$

$2\lambda = (4\lambda)^{\frac{2}{3}}z$

$z = \frac{2\lambda}{(4\lambda)^{\frac{2}{3}}}$

$z = \frac{\sqrt[3]{4}}{2}\sqrt[3]{\lambda}$

$2\left(\frac{\sqrt[3]{4}\cdot\sqrt[3]{\lambda}}{4}\right) + (\sqrt[3]{4\lambda}) + \frac{(\sqrt[3]{4}\sqrt[3]{\lambda})}{2} = 4$

$\lambda = 2$

answer $= \left(\frac{2}{(4\cdot2)^{\frac{2}{3}}},\ \sqrt[3]{4\cdot2},\ \frac{2\cdot2}{(4\cdot2)^{\frac{2}{3}}} \right)$

$$\boxed{= \left(\frac{1}{2},\ 2,\ 1\right)}$$