Sec II

1.) $\int_C yz\,dx + (xz + z)\,dy + (xy + y + 1)\,dz$

over $r(t) = \langle e^{t^2}, t^2e^{t^2}, te^{t^2}\rangle \quad 0 \le t \le 1$

→ when we calculate curl of the function, we get $\langle 0, 0, 0\rangle$, meaning the vector field is conservative, meaning we can use the Fundamental theorem of Line Integrals, or f(end) − f(beginning) to evaluate our answer.

− we must find f, for which grad f = our function

$f_x = yz$, $f_y = xz + z$, $f_z = xy + y + 1$ → $f = xyz + yz + z$

$\overset{\text{start}}{r(0) = (1, 0, 0)}$, $\overset{\text{end}}{r(1) = (e, e, e)}$

$f(e,e,e) - f(1,0,0) = e^3 + e^2 + e - 0 = e^3 - e^2 - e$

my mistake: I did not even notice that I could have checked for the vector field to be conservative, but instead tried to attack the problem by substituting our r values ($r(t) \to x, y, z$) and found dx, dy and dz in terms of t. This got very tedious & long, so I had to stop half way to get on with other problems.

1.a.) $\int_C xe^{xyz}\,dx + ye^{xyz}\,dy + ze^{xyz}\,dz$

looking @ the function we can recognize that this F is ∇f. using properties of derivative of e, we know $f = e^{xyz}$

since $r(t) = \langle t, t^2, t^3\rangle \quad 0 \le t \le 1$

$r(0) = \langle 0, 0, 0\rangle$, $r(1) = \langle 1, 1, 1\rangle$

$f(1,1,1) - f(0,0,0) = e^{(1)(1)(1)} - e^{(0)(0)(0)} = \boxed{e-1}$

(b) $\int_C (4x^3y^2 + 1)\,dx + (2x^4y + 1)\,dy$; $r(t) = \langle \sin t^2, \cos t^3\rangle \quad 0 \le t \le \sqrt{\pi/2}$

$r(0) = (\sin(0), \cos(0)) \to (0, 1)$

greens → $Q_x - P_y = 8x^3y - 8x^3y = \boxed{0}$

= brute force! sub in r(t)

$4(\sin(t)^8 \cos(t)^4)(12\cos t^2 \sin t^2)\,dt + (2\sin(t)^8 \cos t^2)(2\sin(t)^8)\,dt$

for $t = 0$, $\sin(0) = 0$ → whole thing = 0

$t = \sqrt{\frac{\pi}{2}}$, $\cos(\frac{\pi}{2}) = 0$ → whole thing = 0 so, ans. = $\boxed{0}$

2.) $\int_0^5 \int_{(y/5)^{1/3}}^1 \sin x^4 \, dx\,dy$

$x = (\frac{y}{5})^{1/3} \to x^3 = \frac{y}{5} \to 5x^3 = y$

$0 \le x \le 1 \; ; \; 0 \le y \le 5x^3$

$\int_0^1 \int_0^{5x^3} \sin x^4 \, dy\,dx \to$ evaluates to $\sin^4 x \, y \Big|_0^{5x^3} = 5x^3 \sin^4 x$

$\int_0^1 5x^3 \sin^4 x \to -\frac{5\cos(x^4)}{4} \Big|_0^1 \to \boxed{\frac{5}{4}\cos(1) - 1} \approx \boxed{0.5746}$

my mistake: a really dumb algebra mistake w/ a "typo" when writing

2a.) $\int_0^1 \int_0^{e^x} f(x,y)\,dy\,dx$

$y = e^x \to x = \ln y$

$\int_0^e \int_{\ln y}^1 f(x,y)\,dx\,dy$

2b.) $\int_0^{\pi} \int_0^{\sin x} f(x,y)\,dy\,dx$

$y = \sin x \to x = \sin^{-1} y$

$x = 0$

$(0,0), (0,0), (\pi,0) \; (\pi,0)$

$0 \le y \le 1$ $\quad \int_0^1 \int_0^{\sin^{-1} y} f(x,y)\,dx\,dy$

2c.) $\int_0^1 \int_{e^y}^e f(x,y)\,dx\,dy$

$y = 1 \quad y = 0 \quad x = e^y \quad x = e$

$x = e \quad x = e^y \quad y = \ln x$

$\int_1^e \int_0^{\ln x} f(x,y)\,dy\,dx$

3.) ✓

3a.) $x(u,v) = u^2$, $y(u,v) = uv$, $z(u,v) = v^2$

$T_u = (2u, v, 0)$ $\quad T_v = (0, u, 2v)$

$T_u \times T_v = (2v^2, -4uv, 2u^2)$

$\cancel{2v^2 = 1, -4uv = 2, 2u^2 = 4}$ $\quad u^2 = 1 \quad uv = 2 \quad v^2 = 4$

$u = 1, v = 2$

$2(2)^2 i + -4(1)(2), 2(1)^2 = 8i - 8j + 2k$

$8(x-1) + -8(y-2) + 4(z-4) = 0$

$8x - 8y + 4z - 8 + 16 - 16 = 0$

$8x - 8y + 4z = 8$

$$\boxed{z = 2 + 2x - 2y}$$

3b.) $x(u,v) = u^3$ $\quad y(u,v) = v^3$ $\quad z(u,v) = -2uv$

$T_u = (3u^2, 0, -2v)$ $\quad T_v = (0, 3v^2, -2u)$

$T_u \times T_v = (6v^3 \quad 6u^4 \quad 9u^2v^2)$

$u^3 = -1 \quad v^3 = -1 \quad -2uv = 2$

$u = -1 \quad v = -1$

$6(-1)^3 + 6(-1)^4 \quad 9(-1)(-1)^2 = -6i + 6j - 9k$

$-6(x+1) + 6(y+1) + -9(z-2) = 0$

$-6x + 6y - 9z - 6 + 6 + 18 = 0$

$-9z = -18 + 6x - 6y$

$$\boxed{z = 2 + -\frac{6}{9}x + \frac{6}{9}y}$$

4.) $f(x,y,z) = e^{\cos x^2 + \sin xy^2 + \cos z}$

$F: \langle \frac{\partial f}{\partial x}, \frac{\partial f}{\partial y}, \frac{\partial f}{\partial z} \rangle$ (aka divergence!)

$r(t): \langle \cos t, t, \sin t \rangle \quad 0 \le t \le 2\pi$

$r(0): \langle 1, 0, 0 \rangle$

$r(2\pi): \langle 1, 2\pi, 0 \rangle$

$f(1, 2\pi, 0) - f(1, 0, 0) = e^{\cos(1)+1} - e^{\cos(1)+1} = \boxed{0}$

my mistake: tried to blindly brute force the answer again, need to stop and check for easier methods. If the f is clearly some really complicated weird function, brute forcing clearly isn't the answer.

4a.) $f(x,y,z): \sin(x + y^2 + z^3)$

$r(t): \langle t, t^2, t^3 \rangle \quad 0 \le t \le 3$

$\cancel{r(0): (0, 0, 0)}$ Stoke's theorem: $\int_C F \cdot dr = \iint \text{curl} F \cdot dS$

$\cancel{r(3): (3, 9, 27)}$

$\cancel{f(3,9,27) - f(0,0,0):}$

since $F = \text{div} f \rightarrow$ divergence of curl of $f = \boxed{0}$

4b.) $f(x,y) = e^{\cos x + 3\sin y}$

$F = \text{div} f$

using stokes theorem →

div F →

curl of divergence of $f = \boxed{0}$

5.) $\int_R (x^2+y^2+z^2)^3 \, dx\,dy\,dz$

$(x,y,z): x^2+y^2+z^2 \le 1$, $x,y,z \ge 0$

use ~~[illegible]~~ spherical coordinates

$x^2+y^2+z^2 = \rho^2$

$(\rho^2)^3 \rho^2 \sin\varphi \, d\rho\, d\varphi\, d\theta$

$\rho^8 \sin\varphi \, d\rho\, d\varphi\, d\theta$

$\int_0^1 \rho^8 d\rho * \int_0^{\pi/2} \sin\varphi \, d\varphi * \int_0^{\pi/2} d\theta = \frac{1}{9} \cdot 1 \cdot \frac{\pi}{2} = \boxed{\frac{\pi}{18}}$

my mistake: did not immediately recognize spherical coordinates & left blank

5a.) $\int_R (x+y)(x^2+y^2+z^2)^2 \, dx\,dy\,dz$

$(\rho\sin\varphi\cos\theta + \rho\sin\varphi\sin\theta)(\rho^2)^2 \rho^2 \sin\varphi \, d\rho\, d\varphi\, d\theta$

$\rho\sin\varphi(\cos\theta+\sin\theta)\rho^4\rho^2 \sin\varphi \, d\rho\, d\varphi\, d\theta$

$\int_{-2\pi}^0 \int_{-\frac{\pi}{2}}^0 \int_0^1 \rho^7 \sin\varphi(\cos\theta+\sin\theta)\, d\rho\, d\varphi\, d\theta$

$\int_0^1 \rho^7 * \int_{-\frac{\pi}{2}}^0 \sin\varphi \int_{-2\pi}^0 \cos\theta+\sin\theta = \frac{1}{8} \cdot -1 \cdot 0 = \boxed{0}$

5b.) $\int_R z(x^2+y^2+z^2)\, dx\,dy\,dz$

$\rho\sin\varphi(\rho^2) \sin\varphi \, d\rho\, d\varphi\, d\theta$

$\rho^3 \sin\varphi \cdot \sin\varphi \, d\rho\, d\varphi\, d\theta \to \rho^3 \sin^2\varphi \, d\rho\, d\varphi\, d\theta$

$\int_0^1 \rho^3 \cdot \int_0^{\pi} \sin^2\varphi \cdot \int_0^{2\pi} d\theta = \frac{1}{4} \cdot \frac{\pi}{2} \cdot 2\pi = \boxed{\frac{\pi^2}{4}}$

5c.) $\int_R (z-x)\, dx\,dy\,dz$

$\rho = \sqrt{8}$

$(\rho\sin\varphi - \rho\sin\varphi\cos\theta)\rho^2\sin\varphi \, d\rho\, d\varphi\, d\theta$

$(\rho\sin\varphi)(1-\cos\theta)\rho^2\sin\varphi \, d\rho\, d\varphi\, d\theta$

$\rho^3\sin^2\varphi(1-\cos\theta)\, d\rho\, d\varphi\, d\theta$

$\int_0^{2\pi} 1-\cos\theta \int_0^{\pi} \sin^2\varphi \int_0^{\sqrt{8}} \rho^3 = 2\pi \cdot \frac{\pi}{2} \cdot 16 = \boxed{16\pi^2}$

6.) $\int_{-3}^{0}\int_{0}^{\sqrt{9-x^2}} (x^2+y^2)^2\,dy\,dx$

→ convert to polar

$r = 3$

$\frac{\pi}{2} \le \theta \le \pi$ $\quad \int_0^3\int_{\frac{\pi}{2}}^{\pi} (r^2)^2\, r\,dr\,d\theta$

$\int_0^3 r^6 \int_{\frac{\pi}{2}}^{\pi} d\theta = \frac{3^6\pi}{12} = \boxed{\frac{243}{4}\pi}$

my mistake: wrote the problem down incorrectly (x from -3 to 6 instead of -3 to 0.) and did not use polar coordinates

6a.) $\int_{-3}^{0}\int_{0}^{\sqrt{9-x^2}} (x^2+y)\,dy\,dx$

$\int_0^3\int_{\frac{\pi}{2}}^{\pi} (r^2\cos^2\theta + r\sin\theta)\, r\,dr\,d\theta$

$\int_0^3\int_{\frac{\pi}{2}}^{\pi} r^3\cos^2\theta + r^2\sin\theta\,dr\,d\theta$

6b.) $\int_0^4\int_{-\sqrt{16-x^2}}^{0} (x^2+y)\,dy\,dx$

$r: 4$ $\quad \int_0^4\int_{-\frac{\pi}{2}}^{0} r^3\cos^2\theta + r^2\sin\theta\,dr\,d\theta$

θ: $-\frac{\pi}{2} \to 0$

6c.) $\int_{-\frac{\sqrt{2}}{2}}^{\frac{\sqrt{2}}{2}}\int_{x}^{\sqrt{1-x^2}} (x^3+y^2)\,dy\,dx$

$r: 1$

$(r^3\cos^3\theta + r^2\sin^2\theta)\, r\,dr\,d\theta$

$\int_0^1\int_{\frac{\pi}{4}}^{\frac{3\pi}{4}} r^4\cos^3\theta + r^3\sin^2\theta\,dr\,d\theta$

7.)

d.) $\lim_{(x,y)\to(1,1)} \frac{x+y-2}{2x+y-3}$

$$\frac{(1)+(1)-2}{2(1)+(1)-3} = \frac{2-2}{3-3} = \frac{0}{0}$$

if we check both $(t+1, 1)$ and $(1, t+1)$, we do not get a consensus, meaning the ~~overall~~ limit DNE.

my mistake: I tried to find the limit @ a number close to 1, and got $\frac{2}{3}$.

7a.) $\lim_{(x,y)\to(1,3)} \frac{x-1}{y-3}$

$$\frac{1-1}{3-3} = \frac{0}{0}$$

$(t+1, 3) = \frac{t+1-1}{0} =$ NO! $\quad (1, t+3) = \frac{0}{t+3-3} = 0 \rightarrow$ DNE

7b.) $\lim_{(x,y,z)\to(0,0,0)} \frac{x+y+2z}{2x+y+z}$

$$\frac{0+0+2(0)}{2(0)+0+0} = \frac{0}{0}$$

$(1,0,0) = \frac{1+0+0}{2+0+0} = \frac{1}{2}$

$(0,1,0) = \frac{0+1+0}{0+1+0} = 1$ DNE

$(0,0,1) = \frac{0+0+2}{0+0+1} = 2$

8.) ✓

8a.) $f(x,y,z) = xy^2 + yz^2 + z$

$(0,0,0) \to (1,1,-1)$

$r(t) = \langle t, t, -t \rangle$

$r'(t) = \langle 1, 1, -1 \rangle \to ||r'(t)|| = \sqrt{1+1+1} = \sqrt{3}\,dt$

$t t^2 + t t^2 + -t = t^3 + t^3 - t = 2t^3 - t$

$\int_0^1 2t^3 \sqrt{3}\,dt = \sqrt{3} \int_0^1 2t^3 - t\,dt = \frac{t^4 - t^2}{2} \Big|_0^1 = 0 \cdot \sqrt{3} = \boxed{0}$

8b.) $f(x,y) = x + y$

$r = 1$

$\theta = 0$ to π

$(r\cos\theta + r\sin\theta)\, r\, dr\, d\theta \to$

$r^2\cos\theta + r^2\sin\theta\, dr\, d\theta$

$r^2(\cos\theta + \sin\theta)\, dr\, d\theta$

$\int_0^1 r^2 \int_0^{\pi} \cos\theta + \sin\theta = \frac{1}{3} \cdot 2 = \boxed{\frac{2}{3}}$

9.) $F = \langle z, z, x \rangle$

$z = 9 - x^2 - y^2$, $x \geq 0$, $y \geq 0$ $z \geq 0$

$\frac{\partial g}{\partial x} = -2x$ $\frac{\partial g}{\partial y} = -2y$

$\iint (-z(-2x) - z(-2y) + x)\, dA = \iint ((2x+2y)z + x)\, dA$

$\iint (2x+2y)(9-x^2-y^2)\, dA$

$\int_0^3 \int_0^{\sqrt{9-x^2}} ((2x+2y)(9-x^2-y^2)+x)\, dy\, dx$

Using maple $\rightarrow \frac{693}{5} \times -1$ (downwards) $= \boxed{-\frac{693}{5}}$

my mistake: It is somewhat difficult to catch under all my work but I believe I just got caught up in all my work

9a.) $F = \langle x+z, y+z, -x \rangle$

$z = 9 - x^2 - y^2$

$\frac{\partial g}{\partial x} = -2x$ $\frac{\partial g}{\partial y} = -2y$

$(x+z)-2x - -2y(y+z) - x$ $\quad -2x^2 + 2y^2 + (-2x+2y)(9-x^2-y^2) - x$

$-2x^2 - 2xz + 2y^2 + 2yz - x$

$-2x^2 - 2x(9-x^2-y^2) + 2y^2 + 2y(9-x^2-y^2) - x$

$\int_0^3 \int_0^{\sqrt{9-x^2}} -2(x^2-y^2) + -2(x-y)(9-x^2-y^2) - x = \boxed{-9}$

9b.) $\int_0^1 \int_0^1 -2(x^2-y^2) + -2(x-y)(9-x^2-y^2) - x = \boxed{-\frac{1}{6}}$

10.) $x+2y+3z=18$

$f(x,y,z) = xyz$

$g(x,y,z) = x+2y+3z$

$yz = \lambda \quad xz = 2\lambda \quad xy = 3\lambda \quad x+2y+3z=18$

$\frac{yz}{xz} = \frac{\lambda}{2\lambda} = \frac{1}{2} = \frac{y}{x}$

$\frac{zy}{xy} = \frac{\lambda}{3\lambda} = \frac{1}{3} = \frac{z}{x}$

$\rightarrow x+2(\frac{1}{2}x)+3(\frac{1}{3}x) = 18$

$3x = 18$

$x = 6$

$y = 3$

$z = 2$

$\rightarrow \boxed{(6,3,2)}$

my mistake: stupid mistakes w/ algebra

10a.) $f(x,y,z) = xyz$

$2x+y+z=4$

$yz = \lambda 2 \quad xz = \lambda \quad xy = \lambda$

$\frac{yz}{xz} = \frac{\lambda 2}{\lambda} \quad \frac{y}{x} = 2 \quad \frac{xz}{xy} = 1 \quad \frac{z}{y} = 1$

$2(\frac{1}{2}y)+y+y=4$

$y+y+y=4$

$3y=4$

$y = \frac{4}{3}, \frac{2}{3} = x, z = \frac{4}{3} \quad xyz = \frac{4}{3}\cdot\frac{4}{3}\cdot\frac{2}{3}$

$\boxed{\approx 1.185}$

10b.) xy^2z

$2x+y+z=4$

$y^2z = 2\lambda \quad 2yxz = \lambda \quad xy^2 = \lambda$

$2yxz = xy^2 \quad \frac{xy^2}{zy^2} = \frac{\lambda}{2\lambda} = \frac{1}{2} = \frac{x}{z}$

$2yz = y^2$

$2z = y$

$2x = z \quad z+2z+z=4$

$4z=4$

$z=1, x=\frac{1}{2}, y=\frac{1}{2}$

$(\frac{1}{2})(\frac{1}{2})^2(1) = \boxed{\frac{1}{8}}$