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Exam Problem 1: Compute the line integral:

$$\int_C yz\,dx + (xz+z)\,dy + (xy+y+1)\,dz$$

over the path: $r(t) = \langle e^{t^3}, t^2 e^{t^4}, te^{t^7} \rangle$ $\quad 0 < t < 1$

① check to see if the vector field is conservative:

$$\text{curl } F: \begin{vmatrix} i & j & k \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ yz & xz+z & xy+y+1 \end{vmatrix} = (x-x)i-(y-y)j+(z-z)k = \langle 0,0,0 \rangle \therefore \text{ conservative.}$$

② Find potential Function:

$$f(x,y,z) = \int yz\,dx = xyz + g(y,z)$$

$$\frac{\partial}{\partial y}(xyz + g(y,z)) = xz + g_y(y,z) = xz + z \therefore g_y(y,z) = z \qquad g(y,z) = yz + h(z)$$

$$\frac{\partial}{\partial z}(xyz + yz + h(z)) = xy + y + h'(z) = xy + y + 1 \qquad h'(z) = 1$$
$$h(z) = z$$

potential function: $f(x,y,z)$ $xyz + yz + z$

③ Use the fundamental theorem of line integrals:

Starting point: $r(0) = \langle 1,0,0 \rangle$

Ending point: $r(1) = \langle e,e,e \rangle$

$$\int_C yz\,dx + (xz+z)\,dy + (xy+y+1)\,dz = f(r(1)) - f(r(0)) = (e^3+e^2+e)-(0)$$
$$= \boxed{e^3+e^2+e}$$

What I did wrong: In this case I just completly forgot about potential functions. I tried to use the other direct methods but took up all my time because it was too hard. In the future, having a paper with all the formulas in front of me will help me be more prepared for questions like these.

Problem 1a: Compute the line integral:

$$\int_C xe^{xyz}\,dx + ye^{xyz}\,dy + ze^{xyz}\,dz$$

over the path $r(t) = \langle t, t^2, t^3 \rangle$, $0 \leq t \leq 1$

Check to see if it is conservative:

$$\begin{vmatrix} i & j & k \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ xe^{xyz} & ye^{xyz} & ze^{xyz} \end{vmatrix} = xz^2e^{xyz} - xy^2e^{xyz} \leftarrow \text{already not conservative}$$

$F = \langle xe^{xyz}, ye^{xyz}, ze^{xyz} \rangle$

$F(r(t)) = \langle te^{t^6}, t^2e^{t^6}, t^3e^{t^6} \rangle$

$r'(t) = \langle 1, 2t, 3t^2 \rangle$

$F(r(t)) \cdot r'(t) = te^{t^6} + 2t^3e^{t^6} + 3t^5e^{t^6}$

$$\int_0^1 te^{t^6} + 2t^3e^{t^6} + 3t^5e^{t^6}\,dt$$

Problem 1b : Compute the line integral

$$\int_C (4x^3y^2+1)\,dx + (2x^4 y+1)\,dy \qquad \text{over} \quad r(t)\ \langle \sin t^2, \cos t^2 \rangle \qquad 0 \le t \le \sqrt{\pi/2}$$

$$\frac{\partial F_1}{\partial y} = \frac{\partial F_2}{\partial x}$$

$8x^3y = 8x^3y$ ✓ ∴ conservative so we can find a potential function now.

$$f(x,y) = \int 4x^3y^2+1\,dx = 4y^2 \cdot \frac{x^4}{4} + x = x^4y^2 + x + g(y)$$

$$\frac{\partial}{\partial y}\left(x^4y^2+x+g(y)\right) = 2yx^4 + g'(y) = 2x^4y+1 \quad \therefore g'(y)=1 \quad g(y)=y$$

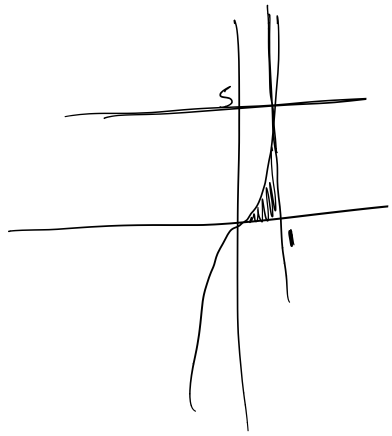
potential function: x^4y^2+x+y

$r(0) = \langle 0,1 \rangle \qquad r(\sqrt{\pi/2}) = \langle 1,0 \rangle$

$f(1,0) - f(0,1) = 1 - 1 = \boxed{0}$

Problem 2: By changing the order of integration, if necessary, evaluate the double integral:

$$\int_0^5 \int_{(y/5)^{1/3}}^{1} \sin x^4 \, dx\, dy$$



$0 \le x \le 1$
$0 \le y \le 5x^3$

$x = 1 \qquad x = \left(\frac{y}{5}\right)^{1/3} \qquad y = 0 \qquad y = 5$

$x^3 = \frac{y}{5}$
$5x^3 = y$

$$\int_0^1 \int_0^{5x^3} \sin x^4 \, dy\, dx$$

$$\int_0^{5x^3} \sin x^4 \, dy = \sin x^4 \cdot y \Big|_0^{5-x^3}$$

$$= 5x^3 \sin x^4$$

$$\int_0^1 5x^3 \sin x^4 \, du$$

$u = x^4 \quad du = 4x^3\, dx$
$x^3 = \frac{1}{4}\, dx$

$$= \int_0^1 5 \cdot \tfrac{1}{4} \sin u \, du = \frac{5}{4} \int_0^1 \sin u \, du$$

$$= \frac{5}{4} (-\cos u)\Big|_0^1 = \frac{5}{4}(-\cos(1) + \cos(0))$$

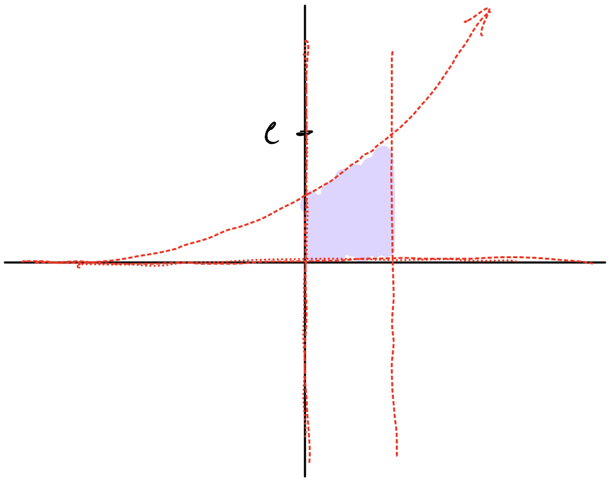
$$\frac{-5\cos(1) + 5}{4}$$

I did not get this qestion wrong.

Problem 2a: Change the order of integration:

$$\int_0^1 \int_0^{e^x} f(x,y)\,dy\,dx$$

$y=0 \quad y=e^x \quad x=0 \quad x=1$



$x = \ln(y)$

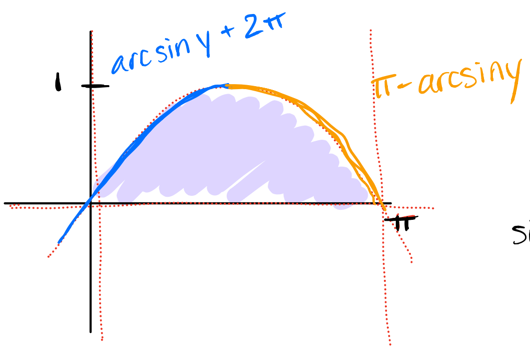
$0 < y < e$

$\ln(y) < x < 1$

$$\boxed{\int_0^e \int_{\ln(y)}^1 f(x,y)\,dx\,dy}$$

Problem 2b: Change the order of integration:

$$\int_0^\pi \int_0^{\sin x} f(x,y)\,dy\,dx$$



$y=0 \quad y=\sin x$

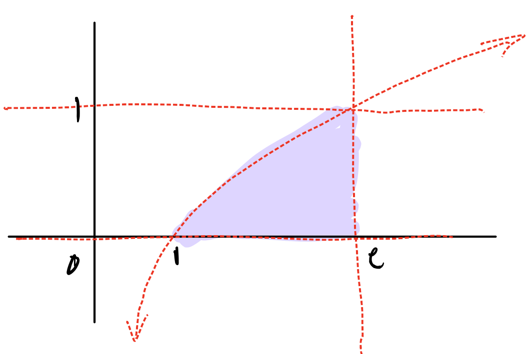
$x=0 \quad x=\pi$

$\sin^{-1} y + 2\pi < x < \pi - \sin^{-1} y$

$0 < y < 1$

$$\boxed{\int_0^1 \int_{\sin^{-1}y + 2\pi}^{\pi - \text{arc}^{-1}\sin y} f(x,y)\,dx\,dy.}$$

Problem 2c: $\int_0^1 \int_{e^y}^{e} f(x,y)\,dx\,dy$



$1 < x < e$

$\ln x < y < e$

$$\boxed{\int_1^e \int_{\ln x}^{e} f(x,y)\,dy\,dx}$$

Exam Problem 3: Find the equation of the tangent plane at the point $(1,1,1)$ to the surface given parametrically by

$$x(u,v) = u^3 v, \quad y(u,x) = uv, \quad z(u,v) = uv^3, \quad -\infty < u < \infty, \quad -\infty < v < \infty$$

$1 = u^3 v$ $\quad uv = 1 \quad uv^3 = 1 \qquad u = 1 \quad v = 1$

$u^3 = \frac{1}{v} \quad u = \frac{1}{v} \quad v^2 = 1$

$v = 1$

$u^3 = u$

$u = 1$

$r = \langle u^3 v, uv, uv^3 \rangle$ $\qquad r_u(1,1) = \langle 3, 1, 1 \rangle$

$\qquad r_v(1,1) = \langle 1, 1, 3 \rangle$

$r_u = \langle 3u^2 v, v, v^3 \rangle$

$r_v = \langle u^3, u, 3v^2 u \rangle$ $\qquad N = \langle 3, 1, 1 \rangle \times \langle 1, 1, 3 \rangle$

$$N = \begin{matrix} i & j & k \\ 3 & 1 & 1 \\ 1 & 1 & 3 \end{matrix} = (3-1)i - (9-1)j + (3-1)k = \langle 2, -8, 2 \rangle = N$$

$\langle 2, -8, 2 \rangle$

$\langle 2, -8, 2 \rangle \cdot \langle x-1, y-1, z-1 \rangle = (2x-2) + (-8y+8) + (2z-2) = 0$

$= 2x - 2 - 8y + 8 + 2z - 2 = 0$

$2x - 8y + 2z + 4 = 0$

$x - 4y + z + 2 = 0$

$z = -x + 4y - 2$

$$\boxed{z = -x + 4y - 2}$$

I did not get this wrong.

Problem 3a: Find the equation of the tangent plane at the point $(1,2,4)$ to the surface given parametrically by:

$x(u,v) = u^2 \quad y(u,v) = uv, \; z(u,v) = v^2, \quad -\infty < u < \infty, \; -\infty < v < \infty$

$u^2 = 1 \quad uv = 2 \quad v^2 = 4$

$u = 1 \quad 1 \cdot v = 2$

$v = 2 \quad v = 2$

$(u,v) = (1,2)$

$r(u,v) = \langle u^2, uv, v^2 \rangle$

$r_u(u,v) = \langle 2u, v, 0 \rangle \qquad r_u(1,2) = \langle 2, 2, 0 \rangle$

$r_v(u,v) = \langle 0, u, 2v \rangle \qquad r_v(1,2) = \langle 0, 1, 4 \rangle$

$$N = \begin{vmatrix} i & j & k \\ 2 & 2 & 0 \\ 0 & 1 & 4 \end{vmatrix} = (8-0)i - (8-0)j + (2-0)k = \langle 8, -8, 2 \rangle$$

$\langle 8,-8,2 \rangle \cdot \langle x-1, y-2, z-4 \rangle = 8(x-1) - 8(y-2) + 2(z-4) = 0$

$= 8x - 8 - 8y - 16 + 2z - 8 = 0$

$8x - 8y + 2z = 32$

$\boxed{4x - 4y + z = 16}$

Problem 3b: Find the equation of the tangent plane at the point $(-1,-1,2)$ to the surface given parametrically by:

$x(u,v) = u^3 \quad y(u,x) = v^3, \; z(u,v) = -2uv, \quad -\infty < u < \infty, \; -\infty < v < \infty$

$u^3 = -1 \quad v^3 = -1 \quad -2uv = 2$

$u = -1 \quad v = -1 \quad -2(-1)(-1) =$

No solution.

$-2 \neq 2$

Exam Problem 4: Let $f(x,y,z) = e^{\cos x^2 + \sin xyz + \cos xz}$ and let,

$F = \langle \frac{\partial f}{\partial x}, \frac{\partial f}{\partial y}, \frac{\partial f}{\partial z} \rangle$

Let C be the curve: $r(t) = \langle \cos t, t, \sin t \rangle \quad 0 \le t \le 2\pi$

$r(0) = \langle 1, 0, 0 \rangle \quad r(2\pi) = \langle 1, 2\pi, 0 \rangle$

$f(1,0,0) = e^{\cos(1) + \sin(0) + \cos(0)} = e^{\cos 1 + 1}$

$f(1,2\pi,0) = e^{\cos 1 + \sin(2\pi) + \cos(0)} = e^{\cos 1 + 1}$

$= e^{\cos(1)+1} - e^{\cos(1)+1} = \boxed{0}$

For this question I failed to realize it involved the fundamental Theorem of line integrals. I tried to solve it directly and ended nowhere. In the future I should be more organized about how I prepare necessary formulas, identities, etc...

Problem 4a: Let $f(x,y,z) = \sin(x + y^2 + z^3)$, and let $F = \langle \frac{\partial f}{\partial x}, \frac{\partial f}{\partial y}, \frac{\partial f}{\partial z} \rangle$

Let C be the curve: $r(t) = \langle t, t^2, t^3 \rangle \quad , \quad 0 \le t \le 3$

We must use the Fundamental Theorem of line integrals:

$r(0) = \langle 0,0,0 \rangle \quad r(3) = \langle 3, 9, 27 \rangle$

$f(0,0,0) = \sin(0) = 0 \quad f(3,9,27) = \sin(3 + 81 + 19683)$

$f(3,9,27) - f(0,0,0) = \sin(19767) - 0 = \boxed{\sin(19767)}$

Problem 4b Let $f(x,y) = e^{\cos x + 3\sin y}$ and let $F = \langle \frac{\partial f}{\partial x}, \frac{\partial f}{\partial y} \rangle$

Let C be the curve: $r(t) = \langle \sin 2t, \cos t \rangle \quad , \quad 0 \le t \le \pi$

We use The Fundamental Theorem of line integrals:

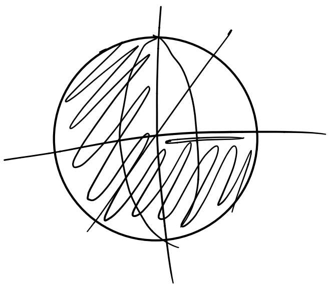
$r(0) = \langle 0, 1 \rangle \quad r(\pi) = \langle 0, -1 \rangle$

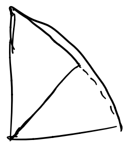
$f(0,1) = e^{\cos(0) + 3(\sin(1))} = e^{1 + 3\sin 1} \quad f(0,-1) = e^{\cos 0 + 3\sin(-1)}$

$f(0,-1) - f(0,1) = \boxed{e^{1+3\sin(-1)} - e^{1+3\sin 1}}$

Exam Problem 5: Evaluate the triple integral:

$$\int_R (x^2+y^2+z^2)^3 \, dx\,dy\,dz$$





$$\int_0^{\frac{\pi}{2}} \int_0^{\frac{\pi}{2}} \int_0^1 (\rho^2)^3 \, \rho^2 \sin\phi \, d\rho \, d\phi \, d\theta$$

$0 \le \rho \le 1$

$0 \le \phi \le \frac{\pi}{2}$

$0 \le \theta \le \frac{\pi}{2}$

$$\int_0^1 \overset{\rho^8}{(\rho^2)^3 \, \rho^2} \sin\phi \, d\rho = \frac{\rho^9}{9} \sin\phi \Big|_0^1 = \frac{\sin\phi}{9}$$

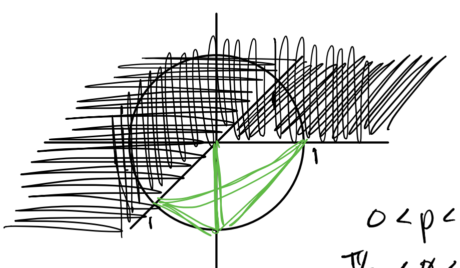
$$\frac{1}{9}\int_0^{\pi/2} \sin\phi \, d\phi = -\cos\phi \Big|_0^{\pi/2} = -\frac{1}{9}(0-1) = \frac{1}{9}$$

$$\int_0^{\pi/2} \frac{1}{9} \, d\theta = \frac{\theta}{9}\Big|_0^{\pi/2} = \frac{\pi/2}{9} = \boxed{\frac{\pi}{18}}$$

I did not get this wrong.

Problem 5a: Evaluate the triple integral:

$$\int_R (x+y)(x^2+y^2+z^2)^2 \, dx\,dy\,dz$$



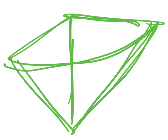
$$\int_{\pi/2}^{\pi} \int_{3\pi/2}^{2\pi} \int_0^1 (\rho\sin\phi\cos\theta + \rho\sin\phi\sin\theta)(\rho^2)^2 \, \rho^2 \sin\phi \, d\rho \, d\theta \, d\phi$$

$$\int_{\pi/2}^{\pi} \int_{3\pi/2}^{2\pi} \int_0^1 \rho^7 \sin^2\phi \, (\cos\theta + \sin\theta) \, d\rho \, d\theta \, d\phi$$

$0 < \rho < 1$

$\pi/2 < \phi < \pi$

$3\pi/2 < \theta < 2\pi$



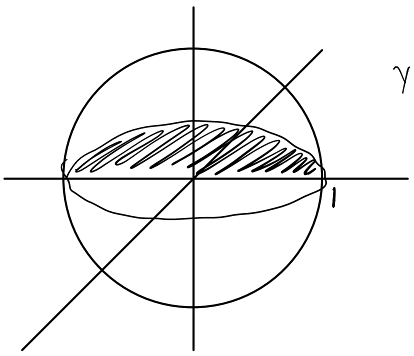
$$\int_0^1 \rho^7 \sin^2\phi \, (\cos\theta + \sin\theta) \, d\rho = \frac{1}{8} \sin^2\phi \, (\cos\theta + \sin\theta)$$

$$\int_{3\pi/2}^{2\pi} \frac{1}{8} \sin^2\phi \, (\cos\theta + \sin\theta) = \frac{1}{8}\sin^2\phi \, (\sin\theta - \cos\theta) \Big|_{3\pi/2}^{2\pi} = (0-1)-(-1-0)$$

$0-1+1+0 = 0$

$$\int_{\pi/2}^{\pi} 0 \, d\phi = 0$$

Problem 5b: $\int_R z(x^2+y^2+z^2)\,dx\,dy\,dz$



$y < 0$

$$\int_0^{\pi}\int_{\pi}^{2\pi}\int_0^{1} p\cos\phi\,(p^2)\sin\phi\;dp\,d\theta\,d\phi$$

$$\int_0^1 p^3\cos\phi\sin\phi\;dp = \frac{p^4}{4}\cos\phi\sin\phi$$

$$\int_{\pi}^{2\pi}\frac{1}{4}\cos\phi\sin\phi\,d\theta = \frac{\theta}{4}\cos\phi\sin\phi\,\Big|_{\pi}^{2\pi}$$

$0 < p < 1$

$\pi < \theta < 2\pi$

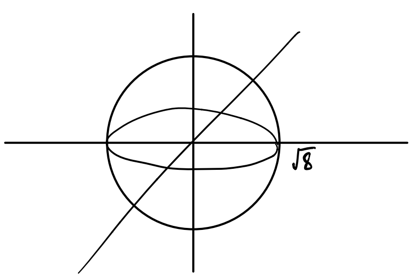
$0 < \phi < \pi$

$$\int_0^{\pi}\frac{\pi}{2}\cos\phi\sin\phi - \frac{\pi}{4}\cos\phi\sin\phi\;d\phi$$

$u = \sin\phi \quad du = \cos\phi\,d\phi$

$$\int_0^0 \sim du - \int_0^0 \sim du = \boxed{0}$$ because of bounds.

Problem 5c: $\int_R (z-x)\,dx\,dy\,dz$



$$\int_0^{\pi}\int_0^{2\pi}\int_0^{\sqrt{8}} (p\cos\phi - p\sin\phi\cos\theta)\,p^2\sin\phi\;dp\,d\theta\,d\phi$$

$$\int_0^{\sqrt{8}} p^3(\cos\phi\sin\phi - \sin^2\phi\cos\theta)\,dp$$

$$= \frac{p^4}{4}(\cos\phi\sin\phi - \sin^2\phi\cos\theta)\Big|_0^{\sqrt{8}}$$

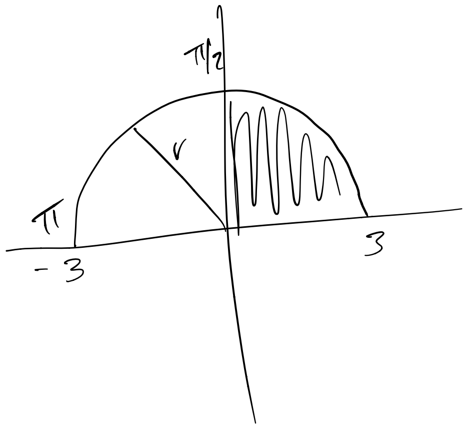
$$16\int_0^{2\pi}\cos\phi\sin\phi - \sin^2\phi\cos\theta\;d\theta = \theta\cos\phi\sin\phi - \sin^2\phi\sin\theta\,\Big|_0^{2\pi}$$

$$16\int_0^{\pi} 2\pi\cos\phi\sin\phi\;d\phi =$$ $u = \sin\phi \quad du = \cos\phi\,d\phi$

$$\int_0^0 \rightarrow \boxed{0}$$

Exam Problem 6:

$$\int_{-3}^{0}\int_{0}^{\sqrt{9-x^2}} (x^2+y^2)^2 \, dy \, dx$$



$$\int_{\frac{\pi}{2}}^{\pi}\int_{0}^{3} (r^2)^2 \, r \, dr \, d\theta$$

$$\int_0^3 r^5 \, dr = \frac{r^6}{6}\Big|_0^3 = \frac{729}{6}$$

$0 \leq r \leq 3$

$\frac{\pi}{2} \leq \theta \leq \pi$

$$\int_{\pi/2}^{\pi} \frac{729}{6} \, d\theta = \frac{729}{6} \theta \Big|_{\pi/2}^{\pi}$$

$$= \frac{729\pi}{6} - \frac{729}{6} \cdot \frac{\pi}{2} = \frac{243\pi}{3} - \frac{243\pi}{12}$$

$$= \frac{729\pi}{12} = \boxed{\frac{243\pi}{4}}$$

I did not get this wrong.

Problem 6a: Convert to polar coordinates, do not evaluate:

$$\int_{-3}^{0}\int_{0}^{\sqrt{9-x^2}} (x^2+y) \, dy \, dx$$

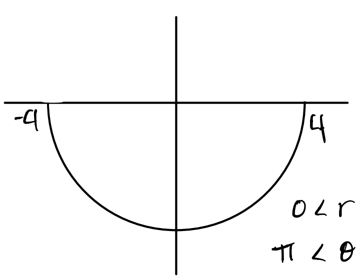
* same bounds as problem above. $x: 3\cos\theta$ $y = 3\sin\theta$

$$\int_{\pi/2}^{\pi}\int_{0}^{3} 9\cos^2\theta + 3\sin\theta \, dr \, d\theta$$

Problem 6b: Convert to polar coordinates, do not evaluate:

$$\int_{0}^{4}\int_{-\sqrt{16-x^2}}^{0} (x^2+y) \, dy \, dx$$

$$\boxed{\int_{\pi}^{2\pi}\int_{0}^{4} 16\cos^2\theta + 4\sin\theta \, dr \, d\theta}$$



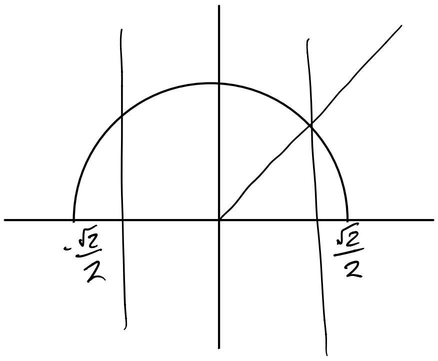
$0 < r < 4$

$\pi < \theta < 2\pi$

Problem 6c:

$$\int_{-\frac{\sqrt{2}}{2}}^{\frac{\sqrt{2}}{2}} \int_{x}^{\sqrt{1-x^2}} (x^3 + y^2)\, dy\, dx$$

$$\int_{\pi/4}^{3\pi/4} \int_{0}^{1} (\cos^3\theta + \sin^2\theta) \cdot r\, dr\, d\theta$$



$0 < r < 1$

ExamProblem 7: Decide whether the following limit exists: If it does find them if not explain why.

(a) (2 points) $\lim_{(x,y)\to(\pi/2,\pi/2)} \frac{\cos x + \sin x}{x+y}$, (b) (2 points) $\lim_{(x,y)\to(0,0)} \frac{x^2-y^2}{x-y}$,

(c) (2 points) $\lim_{(x,y)\to(0,0)} \frac{x-y}{x^2-y^2}$, (d) (4 points) $\lim_{(x,y)\to(1,1)} \frac{x+y-2}{2x+y-3}$,

a) Lets first try plugging in:

$$\frac{\cos(\pi/2)+\sin(\pi/2)}{\frac{\pi}{2}+\frac{\pi}{2}} = \frac{0+1}{\pi} = \boxed{\frac{1}{\pi}}$$

b) We can use an algebra identity:

$(a^2-b^2) = (a+b)(a-b)$

Lets rewrite:

$$\frac{(x+y)\cancel{(x-y)}}{\cancel{x-y}} \to$$

Now we have $\lim_{x,y\to(0,0)} x+y$

We can plug in and get 0.

$0+0 = \boxed{0}$

c) use the same identity from above,

$$\frac{\cancel{x-y}}{(x+y)\cancel{(x-y)}}$$

we have: $\lim_{(x,y)\to(0,0)} \frac{1}{x+y}$ plug in: $\frac{1}{0+0} = \frac{1}{0} =$ DNE

d) if we plug in we get $\frac{0}{0}$. There is also no simplifying to do.

We can compare

$\lim_{(x,y)\to(1+t,1)} \frac{x+y-2}{2x+y-3}$ and $\lim_{(x,y)\to(1,1+t)} \frac{x+y-2}{2x+y-3}$

$$\downarrow$$

$$\frac{1+t+1-2}{2(1+t)+1-3} = \frac{t}{2t} = \frac{1}{2}$$

$2+2t+1-3$

$$\downarrow$$

$$\frac{1+1+t-2}{2+1+t-3} = \frac{t}{t} = 1$$

since $\frac{1}{2} \neq 1$ we can conclude that the limit DNE.

I got a, b, and c correct on the exam but not d because I ran out of time. In the future I need to organize the order in which I will answer questions.

Problem 7a: $\lim_{(x,y)\to(1,3)} \frac{x-1}{y-3}$

plugging in will lead to $\frac{0}{0}$

Compare:

$\lim_{(x,y)\to(1+t,3)} \frac{x-1}{y-3}$ $\quad$ $\lim_{(x,y)\to(1,3+t)} \frac{x-1}{y-3}$

$\downarrow$

$\frac{1+t-1}{3-3}$

Problem 7b

$\lim_{(x,y,z)\to(0,0,0)} \frac{x+y+2z}{2x+y+z}$

Plugging in gets you $\frac{0}{0}$:(

compare:

$\lim_{(x,y,z)\ (t,0,0)} \frac{x+y+2z}{2x+y+z} = \frac{t+0+0}{2t+0+0} = \frac{1}{2}$

$\lim_{(x,y,z)\ (0,t,0)} \frac{x+y+2z}{2x+y+z} = \frac{0+t+0}{0+t+0} = 1$

$\lim_{(x,y,z)\ (0,0,t)} \frac{x+y+2z}{2x+y+z} = \frac{0+0+2t}{0+0+t} = 2$

$\frac{1}{2} \neq 1 \neq 2 \quad \therefore \quad \boxed{\text{DNE.}}$

Exam Problem 8:

8. (10 points) Compute the line integral $\int_C f\,ds$ where

$$f(x, y, z) = xyz$$

and C is the line segment starting at $(0, 0, 0)$ and ending at $(1, 2, -3)$

parametric rep:

$(1-t)(0,0,0) + t(1,2,-3)$

$(0,0,0) + (t, 2t, -3t)$

$r(t) = \langle t, 2t, -3t \rangle$

$r'(t) = \langle 1, 2, -3 \rangle$

$\|r'(t)\| = \sqrt{1+4+9} = \sqrt{14}$

$f(r(t)) = -6t^3$

$\int_0^1 -6t^3 \sqrt{14}\, dt = -6\sqrt{14} \int_0^1 t^3 = \frac{t^4}{4} \Big|_0^1 = \frac{1}{4} - 0$

$= \boxed{-\frac{3\sqrt{14}}{2}}$

I got this question wrong on the exam. I started off correctly and I had the right idea the whole way through but insted of multiplying by $\|r'(t)\|$ I multiplied by $r'(t)$. If I had been more careful I could have gotten the 10 points.

Problem 8a Compute the line integral $\int_C f \cdot ds$ where

$f(x,y,z) = xy^2 + yz^2 + z$ and the line segment starting at $(0,0,0)$ and ending at $(1,1,-1)$

$(1-t)(0,0,0) + t(1,1,-1)$

$r(t) = \langle t, t, -t \rangle$

$r'(t) = \langle 1, 1, -1 \rangle$

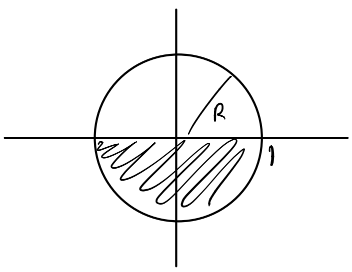
$\|r'(t)\| = \sqrt{1+1+1} = \sqrt{3}$

$f(r(t)) = t(t)^2 + t(-t)^2 - t = t^3 + t^3 - t = 2t^3 - t$

$\sqrt{3}\int_0^1 2t^3 - t = \sqrt{3}\left(2\frac{t^4}{4} - \frac{t^2}{2}\right)\Big|_0^1 = \sqrt{3}\left(\frac{1}{2} - \frac{1}{2}\right) = \boxed{0}$

Problem 8b: Compute the line integral $\int_C f \cdot ds$ where

$f(x,y) = x+y$ and C is the upper circle $\{(x,y) : x^2 + y^2 = 1, y > 0\}$



$0 < r < 1$

$0 < \theta < \pi$

$r(\theta) = \langle \cos\theta, \sin\theta \rangle$ $0 < \theta < \pi$

$r'(\theta) = \langle -\sin\theta, \cos\theta \rangle$

$\|r'(\theta)\| = \sqrt{\sin^2\theta + \cos^2\theta} = \sqrt{1} = 1$

$f(r(\theta)) = \cos\theta + \sin\theta$

$\int_0^\pi \cos\theta + \sin\theta \, d\theta = \sin\theta - \cos\theta \Big|_0^\pi = (0+1) - (0-1)$

$= 0 + 1 - 0 + 1$

$= \boxed{2}$

Exam Problem 9:

9. (10 points) Compute the vector-field surface integral $\int\int_S \mathbf{F}.d\mathbf{S}$ if $\mathbf{F}$ is

$$\mathbf{F} = \langle z, z, x \rangle \quad ,$$

and S is the oriented surface

$$z = 9 - x^2 - y^2 \quad , x \geq 0, y \geq 0, z \geq 0$$

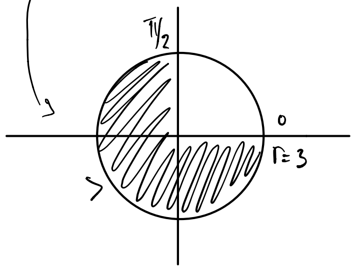
with **downward pointing** normal.

$g(x,y) = 9 - x^2 - y^2$

Find the projection:

$0 = 9 - x^2 - y^2$

$9 = x^2 + y^2$



$P = z \qquad Q = z \qquad R = x$

$$-P\frac{\partial g}{\partial x} - Q\frac{\partial g}{\partial y} + R = -z(-2x) - z(-2y) + x$$

$$= 2zx + 2zy + x$$

$$z(2x+2y) + x$$

$$(9 - x^2 - y^2)(2x+2y) + x$$

convert to polar coordinates:

$$-\int_0^{\pi/2}\int_0^3 (9 - 3\cos^2\theta - 3\sin^2\theta)(6\cos\theta + 6\sin\theta) + 3\cos\theta \quad d\theta$$

This is very long, will use maple instead: $\boxed{-\frac{693}{5}}$

I did not do this problem on the exam. When I first read it I did not know where to start or what formulas to use. More practice on my own time would have helped me on the exam.

Problem 9a:

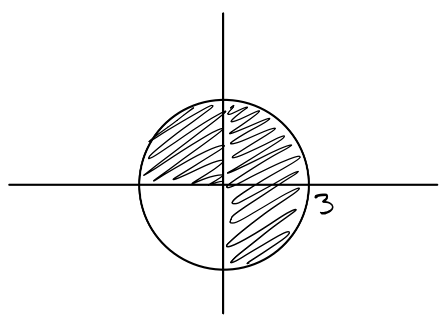
Problem 9a Compute the vector-field surface integral $\int\int_S \mathbf{F}.d\mathbf{S}$ if $\mathbf{F}$ is

$$\mathbf{F} = \langle x + z, y + z, -x \rangle \quad ,$$

and S is the oriented surface

$$z = 9 - x^2 - y^2 \quad , x < 0, y < 0, z \geq 0$$

with **upward pointing** normal.



$x^2 + y^2 = 9$

$g(x,y) = 9-x^2-y^2$

$P = x+z \qquad Q = y+z \qquad R = -x$

$$-(x+z)(-2x) - (y+z)(-2y) - x$$

$$-(-2x^2-2xz) - (-2y^2-2yz) - x$$

$$= 2x^2+2xz+2y^2+2yz-x \rightarrow$$

$$2x(x+z)+2y(y+z)-x$$

$$\int_0^3 \int_0^{\sqrt{9-x^2}} 2x^2+2x(9-x^2-y^2)+2y^2+2y(9-x^2-y^2)-x \; dy\, dx$$

using maple:

```
> int(int(2*x^(2)+2*x*(9-x^2-y^2)+2*y^2+2*y*(9-x^2-y^2)-x, y=0..sqrt(9-x^2)), x=0..3)
;
```

$$\frac{603}{5} + \frac{81\pi}{4}$$

$$\boxed{\frac{603}{5} + \frac{81\pi}{4}}$$

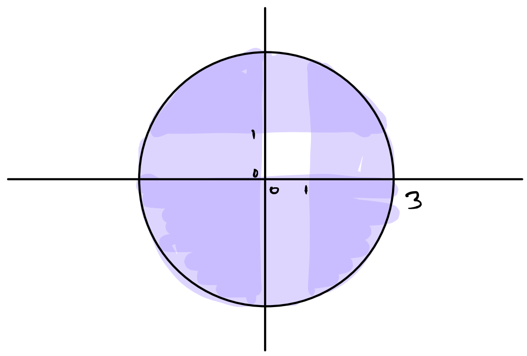
Problem 9b:

Problem 9b Compute the vector-field surface integral $\int\int_S \mathbf{F}.d\mathbf{S}$ if $\mathbf{F}$ is

$$\mathbf{F} = \langle x+z, y+z, -x \rangle \quad ,$$

and S is the oriented surface

$$z = 9 - x^2 - y^2 \quad , 0 < x < 1, 0 < y < 1, z \geq 0$$



$P = x+z \;,\; Q = y+z \;,\; R = -x$

$g(x,y) = 9-x^2-y^2$

Integrand:

$$-(x+z)(-2x) - (y+z)(-2y) - x$$

$$-\int_0^1 \int_0^1 2x^2+2x(9-x^2-y^2)+2y^2+2y(9-x^2-y^2)-x \; dy\, dx$$

maple:

```
> int(int(2*x^(2)+2*x*(9-x^2-y^2)+2*y^2+2*y*(9-x^2-y^2)-x, y=0..1), x=0..1);
```

$$\frac{103}{6}$$

$$\boxed{\frac{103}{6}}$$

Exam Problem 10: Find the POINT on the plane $x+2y+3z=18$ where the function $f(x,y,z)=xyz$ is as large as possible.

We need to use lagrange multipliers.

$\nabla f = \langle yz, xz, xy \rangle \qquad \nabla g = \langle 1,2,3 \rangle$

$$\langle yz, xz, xy \rangle = \lambda \langle 1,2,3 \rangle$$

$yz = \lambda \qquad xz = 2\lambda \qquad xy = \lambda 3 \qquad x+2y+3z=18$

$y = \frac{\lambda}{z} \qquad x\left(\frac{\lambda}{y}\right) = 2\lambda \qquad x\left(\frac{\lambda}{z}\right) = \lambda 3$

$z = \frac{\lambda}{y} \qquad \frac{x\lambda}{y} = \frac{2\lambda}{1} \qquad \frac{x\lambda}{z} = \frac{\lambda 3}{1}$

$2y\lambda = x\lambda \qquad \lambda 3z = x\lambda$

$3z = x$

$x = 2y \qquad z = \frac{x}{3}$

$y = \frac{x}{2}$

$$x + 2\left(\frac{x}{2}\right) + 3\left(\frac{x}{3}\right) = 18$$

$3x = 18$

$x = 6 \longrightarrow y = \frac{6}{2} = 3, \quad z = \frac{6}{3} = 2$

$$\boxed{(6,3,2)}$$

I got this question wrong because I did not finish. I had a good start and recieved some credit but I needed to pace myself better to finish this exam.

Problem 10a

Problem 10a Find the **maximum value** of the function $f(x, y, z) = xyz$ on the plane $2x + y + z = 4$

$\nabla f = \langle yz, xz, xy \rangle \qquad \nabla g = \langle 2, 1, 1 \rangle$

$\langle yz, xz, xy \rangle = \lambda \langle 2, 1, 1 \rangle$

$yz = 2\lambda, \quad xz = \lambda, \quad xy = \lambda \qquad 2x + y + z = 4$

$\frac{xz}{1} = \frac{yz}{2}$

$\frac{\cancel{x}z = \lambda}{\cancel{x}y = \lambda} = \frac{z}{y} = 1$

$\frac{y}{z} = 1$

$2x\cancel{z} = y\cancel{z}$

$2x = y$

$x = \frac{y}{2}$

$2\left(\frac{y}{2}\right) + 2y = 4$

$3y = 4$

$y = 4/3, \quad z = 4/3, \quad x = \frac{4}{6} = \frac{2}{3}$

$f\left(\frac{2}{3}, 4/3, 4/3\right) = \frac{2}{3} \cdot \frac{4}{3} \cdot \frac{4}{3} = \boxed{\frac{32}{27}}$

Problem 10b:

Problem 10b Find the point on the plane $2x + y + z = 4$ where $f(x, y, z) = xy^2z$ is as large as possible. (You can use Maple)

$\nabla f = \langle y^2z, 2yxz, xy^2 \rangle \qquad g\nabla = \langle 2, 1, 1 \rangle$

$\langle y^2z, 2yxz, xy^2 \rangle = \lambda \langle 2, 1, 1 \rangle$

$y^2z = 2\lambda \qquad 2yxz = \lambda, \qquad xy^2 = \lambda$

$\cancel{4}\cancel{z}^3 = \cancel{4}xy\cancel{z} \qquad 2\cancel{y}\cancel{x}z = \cancel{x}\cancel{y}^2$

$1 = xy \qquad 2z = y$

$x = \frac{1}{y} \qquad z = \frac{y}{2}$

plug in

$2\left(\frac{1}{y}\right) + y + \frac{y}{2} = 4$

$\frac{2}{2y} + y + \frac{y}{2} = 4$

Using maple, $y = \frac{4}{3} + \frac{\sqrt{10}}{3}$

$$x = \frac{1}{\frac{4}{3} + \frac{\sqrt{10}}{3}} = \frac{3}{4+\sqrt{10}}$$

$$z = \frac{\frac{4}{3} + \frac{\sqrt{10}}{3}}{2} = \frac{6}{4+\sqrt{10}}$$

$$\boxed{\text{point : } \left(\frac{3}{4+\sqrt{10}}, \frac{6}{4+\sqrt{10}}, \frac{4+\sqrt{10}}{3} \right)}$$